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A model independent spin analysis of fundamental particles using azimuthal asymmetries

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ABSTRACT: Exploiting the azimuthal angle dependence of the density matrices we construct observables that directly measure the spin of a heavy unstable particle. A novelty of the approach is that the analysis of the azimuthal angle dependence in a frame other than the usual helicity frame offers an independent cross-check on the extraction of the spin. Moreover, in some instances when the transverse polarisation tensor of highest rank is vanishing, for an accidental or dynamical reason, the standard azimuthal asymmetries vanish and would lead to a measurement with a wrong spin assignment. In a frame such as the one we construct, the correct spin assignment would however still be possible. The method gives direct information about the spin of the particle under consideration and the same event sample can be used to identify the spins of each particle in a decay chain. A drawback of the method is that it is instrumental only when the momenta of the test particle can be reconstructed. However we hope that it might still be of use in situations with only partial reconstruction. We also derive the conditions on the production and decay mechanisms for the spins, and hence the polarisations, to be measured at a collider experiment. As an example for the use of the method we consider the simultaneous reconstruction, at the partonic level, of the spin of both the top and the W in top pair production in e^+e^- in the semi-leptonic channel.

KEYWORDS: Spin, Quantum interference.

Contents

1. Introduction

The Standard Model (SM) of particle physics has been successful in explaining all the collider observables till date with a high degree of precision. This is remarkable considering that the particle content of the model is not complete since it requires a scalar spin-less particle, the Higgs. In the SM formulation, this particle is an integral ingredient of the mechanism of electro-weak (EW) symmetry breaking. This mechanism is still not well understood. For example, the fact that in the SM no symmetry protects the mass of the spin-less Higgs poses the hierarchy problem. Any solution to these issues brings in new particles and interactions at TeV scale with varying spin and gauge quantum number assignment. In a collider experiment, where almost all these particles are expected to be produced and decay to the light SM particles, the gauge quantum numbers, in principle, can be re-constructed by adding-up the gauge quantum numbers of all the observed light SM particles. Spin, on the other hand, shows up only in the distributions in various kinematic variables in the production and the decay sub-processes. Since the knowledge of the spin, along with the gauge quantum numbers, can enable us to distinguish amongst various candidate theories of physics beyond the SM (BSM) there has been growing interest in this subject recently in the context of the upcoming Large Hadron Collider (LHC) [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] and also in the context of proposed International Linear Collider (ILC) [20, 21, 22, 23, 24, 25].

Most of the BSM models have new particles that are partners of the SM particles based on their gauge quantum numbers assignments. However, the new particles may differ in the spin assignments. In models with supersymmetry (SUSY), the spin of the SM partner differs by $\frac{1}{2}$ owing to the fermionic nature of the SUSY generators. There are however many other models such as UED where the spin of the partner is same as in the SM. In both kind of models a Z_2 symmetry can be left over, leading to a heavy stable particle in the spectrum which can be the dark matter candidate. In SUSY models, the lightest neutralino, singlino, gravitino or axino can be stable, while in the models with universal extra dimensions (UED) the first Kaluza-Klein (KK) excitation of photon can be stable and are the dark matter candidate. These dark matter candidates can not be detected directly in collider experiments. Thus if these particles appear at the end of a decay chain, the re-construction of spin can be non-trivial, specially at a hadron collider like LHC. Moreover it would be also important to infer the spin, and other properties, of these dark matter candidates since these properties are important for the indirect detection of dark matter in astrophysical experiments.

The spin of a (new) particle determines the Lorentz structure of its couplings with the other SM fermions and bosons. This, in a way, fixes its dominant production and decay mechanisms. In many cases a careful study of the energy dependence of the cross section around threshold can distinguish between spin–0 and spin–$\frac{1}{2}$ particles

for example. Other methods to determine spin involve decay particles correlators. At the heart of these more direct methods is the decay helicity amplitude. For example, the helicity amplitude of a particle with spin–s and helicity λ with $-s \le \lambda \le s$ decaying into two particles of spins s_1 and s_2, with helicity $l_{1,2}$ respectively, can be written as [26]

$$M^{s\lambda}_{l_1 l_2}(\theta,\phi) = \sqrt{\frac{2s+1}{4\pi}} D^{s*}_{\lambda l}(\phi,\theta,-\phi)\mathcal{M}^s_{l_1,l_2} = \sqrt{\frac{2s+1}{4\pi}} e^{i(\lambda-l)\phi} d^s_{\lambda l}(\theta)\mathcal{M}^s_{l_1,l_2}, \;\; l = l_1 - l_2. \tag{1.1}$$

Here $\mathcal{M}^s_{l_1,l_2}$ is the reduced matrix element. This has been written most conveniently in the rest frame of the decaying particle. In fact the helicity here is the projection of the spin on the quantisation axis. The polar angle θ is measured w.r.t. this quantisation axis and the azimuthal angle ϕ is measured around the same quantisation axis with freedom to chose the $\phi = 0$ plane. In most of the examples, it is useful to chose the production plane of the decaying particle as the $\phi = 0$ plane. Boosting along the quantisation axis will leave the value of the helicity unchanged. The angular distribution in these angles encodes the spin information through the rotation matrix D which *factorises* into an overall phase factor $e^{i(\lambda-l)\phi}$ carrying the azimuthal angle ϕ dependence and the the $d^s_{\lambda l}$ function carrying the polar angle dependence. The latter can be expressed as [27]

$$\begin{aligned} d^s_{\lambda l}(\theta) = & \sum_t (-1)^t \, \frac{[(s+\lambda)!\,(s-\lambda)!\,(s+l)!\,(s-l)!]^{1/2}}{t!\,(s+\lambda-t)!\,(s-l-t)!\,(t+l-\lambda)!} \\ & \times \left(\cos\frac{\theta}{2}\right)^{2(s-t)+\lambda-l} \left(\sin\frac{\theta}{2}\right)^{2t+l-\lambda} \end{aligned} \tag{1.2}$$

with $-s \le \lambda, l \le s$. The sum is taken over all values of t which lead to non negative factorials. The differential rates have therefore polynomial dependence on $\cos\theta$ up to degree $2s$ and the azimuthal modulation coming from the off-diagonal elements of the density matrix ranges up to $\cos(2s\phi)$. One can construct observables to extract the degree of $\cos\theta$ and/or $\cos\phi$ distribution. If the highest mode for, say, the azimuthal dependence $\cos(2s\phi)$ can be extracted this would be an unambiguous measure of the spin, s of the particle.

Other methods have been used or advocated to determine spin.

1. Exploiting the behaviour of the total cross-section at threshold for pair production [15, 22] or the threshold behaviour in the off-shell decay of the particle [1],

2. distribution [4, 7, 11, 13, 22] in the production (polar) angle relying on a known production mechanism,

3. comparing different spin assignments to intermediate particles in a process for a given collider signature [1, 5, 7, 8, 9, 10, 17, 18, 19],

4. comparing SUSY vs UED for a given collider signature [3, 4, 6, 11, 12, 14, 16, 21, 23, 24, 25],

5. extracting the $(\cos\theta)^{2s}$ polar angle dependence [2, 3, 7, 8, 14, 20, 22, 28, 29] or $\cos 2s\phi$ azimuthal angle dependence [23, 24, 25, 29] of the decay distributions.

The first four methods are indirect ways to assess the spin information subject to some assumptions and can only support or falsify a hypothesis. For example, the threshold behaviour depends not only on the spin but the parity of the particle as well [1] and for a particle of given spin it could be used to determine its parity [30]. Further, it has been shown [22] that for pair production in an e^+e^- annihilation, the threshold behaviour alone can not determine the spin of the particle. With β the velocity of the produced particle in the laboratory, at threshold the cross section for a scalar scales as β^3 behaviour while for a spin–1/2 it goes like β, except for Majorana fermions which can have β^3 behaviour. Note that these β^n characteristics do not take into account Sommerfeld/Coulomb[31] type corrections. The spin–1 particle can also have threshold behaviour and production angle dependence same as that of fermions with only difference coming from the distributions of their decay products. Thus, threshold behaviour and production angle distributions can at best be used only to confirm the spin assignment not to determine it. In the second method, one usually assumes a production topology, like for example s–channel pair production through a gauge boson. In this method the production angle dependence will depend upon the spin of the test particle. But still this dependence is not unique and can be obtained for higher spin test particles.

The third and the fourth method uses numerical values of correlators or differences in the distributions, which can be modified by the changes in the couplings or the presence of addition particles in virtual exchange *etc.* Thus, one can not use this method without having re-constructed the spectrum of the theory experimentally. The last method, which uses decay correlators, gives either the spin of the particle or the *absolute lower limit* on its spin. We note that the moments of the polar angle distribution discussed in Ref.[28] gives an *upper limit* on the spin of the particle.

1.1 Spin through the polar angle

Earlier studies of spin measurements used the average values of $\cos\theta$ or angular asymmetry, with appropriately defined polar angle θ, in the process of 2-body decay [28] or cascade decay [29]. The numerical values of the angular asymmetries or the moments of angular distribution gave estimates of the spin of the decaying particles in a model independent way. Most of the recent spin studies using decay kinematics focus on a decay chain that can be realised in SUSY or UED models. All the intermediate particles in the decay chains are assumed on-shell such that there is no distortions coming from the shape of the off-shell propagator and that it can be decomposed as

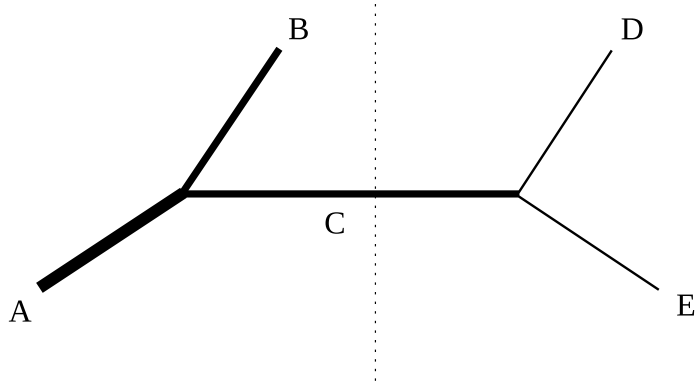

Figure 1: The typical decay chain studied for the spin analysis of particle A via its decay into observed particles B and D and a missing particle E.

a series of two body decays, simplifying the calculations. For example, we look at a 3–body decay chain of a particle A, shown in Fig.1. We look in the rest frame of the intermediate particle C whose spin is to be determined. Using crossing symmetry we write the matrix element for the s–channel process $AB \to C \to DE$ as [26]

$$M^{\lambda_A,\lambda_B}_{\lambda_D,\lambda_E}(\theta_{BD},\phi) = (2s+1)\ \ d^s_{\lambda_i,\lambda_f}(\theta_{BD})\ \ e^{i\phi(\lambda_i-\lambda_f)}\ \ \mathcal{M}^s_{\lambda_i,\lambda_f}, \tag{1.3}$$

where, $\lambda_i = \lambda_B - \lambda_A$ and $\lambda_f = \lambda_D - \lambda_E$. The rotation matrix $d^s_{\lambda_i,\lambda_f}(\theta)$ for spin s particle is $2s$ degree polynomial in $\cos(\theta/2)$ and $\sin(\theta/2)$, Eq.(1.2), which on squaring transforms to a $2s$ degree polynomial in $\cos\theta$. This leads to a $2s$ degree polynomial form of the angular distribution as

$$\frac{d\Gamma(A \to BDE)}{d\cos\theta_{BD}} = Q_0 + Q_1\ \cos\theta_{BD} +\ \ldots\ + Q_{2s}\ \cos^{2s}\theta_{BD}. \tag{1.4}$$

Thus, we see that the degree of these polynomials is a consequence of the representation of the particle under Lorentz or rotation group, in other words, the spin of the particle, provided C is produced on-shell, i.e. $(p_D + p_E)^2 = p_C^2 = m_C^2 =$ constant.

One can also describe the decay in powers of some invariants, the highest power giving a measure of spin. Indeed, with $m^2_{BD} = (p_B + p_D)^2$ we can write

$$\frac{d\Gamma(A \to BDE)}{dm^2_{BD}} = P_0 + P_1\ m^2_{BD} +\ \ldots\ + P_{2s}\ (m^2_{BD})^{2s}, \tag{1.5}$$

obtained from Eq.(1.4) through $dm^2_{BD} = 2E_B E_D \beta_B \beta_D d\cos\theta_{BD}$ by using a transformation of variables. Note that we could have $P_{2s} = 0$ and $P_{2s-1} \neq 0$ for some kinematical or dynamical reason, in this case we would set the lower limit on the spin to be $s - \frac{1}{2}$. We note that the above method involves two decay products of particle A while it measures the spin of the intermediate particle C and not the spin of the mother particle A. To directly measure the spin of A, we need to use the polar angle of every decay products w.r.t. the quantisation axis of A. The distribution w.r.t this decay angle looks identical to Eq.(1.4) with s being the spin of A.

1.2 Spin through azimuthal angle

Another method of direct spin re-construction is to use the azimuthal angle distribution of the decay product about the quantisation axis of the decaying particle A. This is the main thrust of the present work. Using the form of the rotation matrices it can be shown, see later, that the azimuthal distribution appearing from the interference of different helicity states, has the general form

$$\frac{d\Gamma}{d\phi} = a_0 + \sum_{j=1}^{2s} a_j \ \cos(j\phi) + \sum_{j=1}^{2s} b_j \ \sin(j\phi), \tag{1.6}$$

with a_j being the CP even contributions while the b_j being CP odd contributions. A statistically significant non-zero value of a_{2s}/a_0 or b_{2s}/a_0 proves the particle spin to be s. The coefficients, a_j and b_j, depend on the dynamics of production and decay processes and we will see that they are proportional to the degree of quantum interference of different helicity states of the particle A, or in other words, to the off-diagonal elements of production and decay density matrices. This distribution (but for the CP odd part) has been proposed in [23] and used in [24] to measure the spin of W and Z bosons at LEP-II and Tevatron, respectively. The azimuthal distribution in the laboratory frame is not simple sin or cos, however it is sensitive to the polarisation of the decaying particle as shown in Ref. [32]. In this paper, we study the azimuthal distribution, Eq.(1.6), in a model independent way to determine the constants a_js and b_js in terms of production and decay mechanism and construct collider observables to possibly measure these constants. We construct the observables in two different frames of reference and compare their merits.

This paper is organised as follows. In section 2 we give the angular distribution of decay products for a general process of production and decay with emphasis on the case of spin–$\frac{1}{2}$ and spin–1 particles. We describe the azimuthal distribution in terms of observables (asymmetries) to be used at colliders or event-generators in section 3. A numerical example of top quark decay chain is given in section 4 for the two different reference frames. We conclude in section 5. Additional expressions are given in the appendices.

2. Density matrices, polarisation and azimuthal distributions

To assess the spin of an unstable particle A, we look at a general n–body production process $B_1 B_2 \to A \ A_1 \ ... \ A_{n-1}$ followed by the decay of A as $A \to BC$, for example. The other particles A_i's produced in association with A can be either stable or decay inclusively. The differential rate for such a process is given by (see for example [32]),

$$d\sigma = \sum_{\lambda,\lambda'} \left[\frac{(2\pi)^4}{2I} \rho(\lambda,\lambda') \delta^4 \left(k_{B_1} + k_{B_2} - p_A - \left(\sum_i^{n-1} p_i \right) \right) \frac{d^3 p_A}{2E_A (2\pi)^3} \ \prod_i^{n-1} \frac{d^3 p_i}{2E_i (2\pi)^3} \right]$$

$$\times \left[\frac{1}{\Gamma_A} \frac{(2\pi)^4}{2m_A} \Gamma'(\lambda,\lambda')\delta^4(p_A - p_B - p_C) \frac{d^3p_B}{2E_B(2\pi)^3} \frac{d^3p_C}{2E_C(2\pi)^3} \right] \tag{2.1}$$

after using the narrow-width approximation for the unstable particle A, thereby factoring out the production part from the decay. Here we have $I^2 = [m^2_{B_1B_2} - (m_{B_1} + m_{B_2})^2][m^2_{B_1B_2} - (m_{B_1} - m_{B_2})^2]$, $m^2_{B_1B_2} = (k_{B_1} + k_{B_2})^2$, Γ_A is the total decay width of A, m_A is the mass of A and $\Gamma_A << m_A$. The production and decay density matrices for A are denoted by $\rho(\lambda,\lambda')$ and $\Gamma'(\lambda,\lambda')$, respectively. The terms in square brackets in Eq.(2.1) are Lorentz invariant combinations. The phase space integration can be performed in any frame of reference without loss of generality.

Since we are interested in the decay distribution of A, we perform the phase space integrations in the rest frame A. We integrate the first square bracket in Eq.(2.1) and denote it as

$$\sigma(\lambda,\lambda') = \int \frac{(2\pi)^4}{2I} \rho(\lambda,\lambda')\delta^4\left(k_{B_1} + k_{B_2} - p_A - \left(\sum_i^{n-1} p_i\right)\right) \frac{d^3p_A}{2E_A(2\pi)^3} \prod_i^{n-1} \frac{d^3p_i}{2E_i(2\pi)^3} \,. \tag{2.2}$$

We note that the total integrated production cross-section, without cuts, of the process is given by the sum of diagonal terms $\sigma_A = \text{Tr}\ \sigma(\lambda,\lambda')$, while the off-diagonal terms of $\sigma(\lambda,\lambda')$ denote the production rates for transverse/tensor polarisation states or, in other words, for the quantum interference states. Further, we rewrite $\sigma(\lambda,\lambda') = \sigma_A\ P_A(\lambda,\lambda')$, where $P_A(\lambda,\lambda')$ is the polarisation density matrix for A in the corresponding production process. Similarly, we can partially integrate the second term in Eq.(2.1) and write it as

$$\begin{aligned} \int \frac{1}{\Gamma_A} \frac{(2\pi)^4}{2m_A} \Gamma'(\lambda,\lambda')\delta^4(p_A - p_B - p_C) \frac{d^3p_B}{2E_B(2\pi)^3} \frac{d^3p_C}{2E_C(2\pi)^3} \\ = \frac{B_{BC}(2s+1)}{4\pi}\ \Gamma_A(\lambda,\lambda')d\Omega_B, \end{aligned} \tag{2.3}$$

where B_{BC} is the branching ratio for the decay $A \to BC$, s is spin of A, $\Gamma_A(\lambda,\lambda')$ is the decay density matrix normalised to unit trace, $d\Omega_B$ is the solid angle measure for the decay product B.[1] Combining Eq.(2.2) and Eq.(2.3) in Eq.(2.1) we get the decay angular distribution as

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega_B} = \frac{2s+1}{4\pi} \sum_{\lambda,\lambda'}\ P_A(\lambda,\lambda')\ \ \Gamma_A(\lambda,\lambda'), \tag{2.4}$$

where $\sigma = \sigma_A\ B_{BC}$, the total cross-section for production of A followed by its decay into BC state. The polarisation density matrix contains the dynamics of the production process and we will discuss its form for spin–$\frac{1}{2}$ and spin–1 particle in the

[1] One can also consider 3–body or higher body decay of A in Eq.(2.1) and write Eq.(2.3) by integrating all the phase space except Ω_B. One example of this will be the top-quark decay [32].

following sections.

First we will discuss the general structure of the decay density matrix which can be studied independently of the production mechanism. The decay density matrix for a spin–s particle, expressed in terms of helicity amplitudes Eq.(1.1), is given by

$$
\begin{aligned}
\Gamma'^s(\lambda,\lambda') &= \sum_{l_1,l_2} M^{s\lambda}_{l_1 l_2} M^{*s\lambda'}_{l_1 l_2} \\
&= \left(\frac{2s+1}{4\pi}\right) e^{i(\lambda-\lambda')\phi} \sum_{l_1,l_2} d^s_{\lambda l}(\theta) d^s_{\lambda' l}(\theta)\ |\mathcal{M}^s_{l_1,l_2}|^2 \\
&= e^{i(\lambda-\lambda')\phi} \sum_{l} d^s_{\lambda l}(\theta) d^s_{\lambda' l}(\theta) \left[\sum_{l_1} \left(\frac{2s+1}{4\pi}\right) |\mathcal{M}^s_{l_1,l_1-l}|^2\right] \\
&= e^{i(\lambda-\lambda')\phi} \sum_{l} d^s_{\lambda l}(\theta) d^s_{\lambda' l}(\theta)\ a^s_l,
\end{aligned}
\tag{2.5}
$$

where

$$
a^s_l = \left(\frac{2s+1}{4\pi}\right) \sum_{l_1} |\mathcal{M}^s_{l_1,l_1-l}|^2, \qquad |l_1| \le s_1,\ \ |l_1 - l| \le s_2,\ \ |l| \le s. \tag{2.6}
$$

For a spin s particle there are $2s+1$ different a^s_l's that define the decay density matrix with

$$
\mathrm{Tr}(\Gamma'^s(\lambda,\lambda')) = \sum_l a^s_l\ .
$$

Dividing the Γ'^s by its trace leaves us with $2s$ independent quantities involving a^s_l's to define the normalised decay density matrix of a spin–s particle. Now, the normalised decay density matrix can be written as

$$
\Gamma_A(\lambda,\lambda') = e^{i(\lambda-\lambda')\phi}\ \frac{\sum_l d^s_{\lambda l}(\theta) d^s_{\lambda' l}(\theta) a^s_l}{\sum_l a^s_l} = e^{i(\lambda-\lambda')\phi}\ \gamma_A(\lambda,\lambda';\theta), \tag{2.7}
$$

where $\gamma_A(\lambda,\lambda';\theta) \equiv \gamma_A(\lambda,\lambda')$ is the *reduced* normalised decay density matrix with only θ dependence left. It is important to keep in mind that the ϕ dependence is an overall phase and we see clearly that the differential cross section will have a more transparent dependence on the azimuthal angle than the polar angle. Using the relation in Eq.(2.7) we can re-write Eq.(2.4) as

$$
\begin{aligned}
\frac{1}{\sigma}\frac{d\sigma}{d\Omega_B} = \frac{2s+1}{4\pi} \Bigg[& \sum_{\lambda} P_A(\lambda,\lambda)\ \ \gamma_A(\lambda,\lambda) \\
& + \sum_{\lambda\neq\lambda'} \Re[P_A(\lambda,\lambda')]\ \ \gamma_A(\lambda,\lambda')\ \cos((\lambda-\lambda')\phi) \\
& - \sum_{\lambda\neq\lambda'} \Im[P_A(\lambda,\lambda')]\ \ \gamma_A(\lambda,\lambda')\ \sin((\lambda-\lambda')\phi) \Bigg],
\end{aligned}
\tag{2.8}
$$

which is similar to Eq.(1.6) after integrating out $\cos\theta$. Thus we have a simple looking ϕ distribution of the decay product and also the coefficients of the different harmonics in the distribution. We emphasise again that the ϕ dependence enters only through terms with $\lambda \neq \lambda'$, in other words the off-diagonal elements of the production and decay density matrices. When integrating over the full space without any cuts, the information contained in these terms will be lost. Another point to stress is that the form of the distribution Eq.(2.8) remains same in any other frame as long as ϕ is measured around the momentum axis of the particle with some suitable reference for $\phi = 0$. The measurement of the $\cos n\phi$ (with $n \leq 2s$) modulation that stems from the part describing the decay depends on the size of the corresponding factor $P_A\gamma_A$ which is controlled by the interactions of particle A. The factors P_A describe the different polarisations with which the particle is produced and the factor γ_A depends on the dynamics controlling the decay. One of the aims of this paper is to analyse how one can use this understanding to maximise these modulations and especially the modulation with $\cos 2s\phi$ which is the most unambiguous measure of the spin-s of the decaying particle. For illustration and as a guide, in the following, we take a close look at the production and decay density matrices for spin–$\frac{1}{2}$ and spin–1 particles to identify the conditions on the production and decay dynamics for the spin to be measured.

2.1 Spin–$\frac{1}{2}$ particle

For the decay of spin–$\frac{1}{2}$ particle, $|\frac{1}{2}, l\rangle \to |s_1, l_1\rangle + |s_2, l_2\rangle$, the normalised decay density matrix, in the rest frame or rather the *helicity rest frame* [33], can be written as

$$\Gamma_{\frac{1}{2}}(\lambda,\lambda') = \begin{bmatrix} \frac{1+\alpha\cos\theta}{2} & \frac{\alpha\sin\theta}{2}\, e^{i\phi} \\ \frac{\alpha\sin\theta}{2}\, e^{-i\phi} & \frac{1-\alpha\cos\theta}{2} \end{bmatrix}, \tag{2.9}$$

using Eq.(2.5). Here $\alpha = (a^{1/2}_{1/2} - a^{1/2}_{-1/2})/(a^{1/2}_{1/2} + a^{1/2}_{-1/2})$ and a^j_l are defined in terms of reduced matrix elements in Eq.(2.6) and for spin–$\frac{1}{2}$ particles given as

$$\begin{aligned} a^{1/2}_{1/2} &= \left(\frac{1}{2\pi}\right)\sum_{l_1} |\mathcal{M}^{1/2}_{l_1,l_1-1/2}|^2 \qquad |l_1| \leq s_1, \quad |l_1 - 1/2| \leq s_2 \\ a^{1/2}_{-1/2} &= \left(\frac{1}{2\pi}\right)\sum_{l_1} |\mathcal{M}^{1/2}_{l_1,l_1+1/2}|^2 \qquad |l_1| \leq s_1, \quad |l_1 + 1/2| \leq s_2. \end{aligned} \tag{2.10}$$

The explicit calculation of α for the spin–$\frac{1}{2}$ particle decaying into two body final state is given in the Appendices C.1 and C.2 for decays into a lighter spin–1/2 and either a scalar or spin–1 considering general effective operators. We have restricted ourselves to operators of dimension 4. It can be seen that α is zero for a pure vector

or pure axial-vector coupling in the case of decay to a spin-1. It can also be small depending on the masses of the daughter particles.

The polarisation density matrix for a spin–$\frac{1}{2}$ particle can be parameterised as

$$P_{\frac{1}{2}}(\lambda,\lambda') = \frac{1}{2}\begin{bmatrix} 1+\eta_3 & \eta_1 - i\eta_2 \\ \eta_1 + i\eta_2 & 1-\eta_3 \end{bmatrix}, \tag{2.11}$$

where η_1 is the transverse polarisation of A in the production plane, η_2 is the transverse polarisation of A normal to the production plane and η_3 is the average helicity or polarisation along the momentum of A or polarisation along the quantisation axis. Combining the expression of $\Gamma_{\frac{1}{2}}(\lambda,\lambda')$ and $P_{\frac{1}{2}}(\lambda,\lambda')$ in Eq.(2.8) we get the angular distribution of spin–$\frac{1}{2}$ particle as [32]

$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\Omega_B} = \frac{1}{4\pi}\left[1+\alpha\eta_3\cos\theta + \alpha\eta_1\sin\theta\cos\phi + \alpha\eta_2\sin\theta\sin\phi\right]. \tag{2.12}$$

The $\cos\theta$ averaged azimuthal distribution is given by

$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\phi} = \frac{1}{2\pi}\left[1+\frac{\alpha\eta_1\pi}{4}\cos\phi + \frac{\alpha\eta_2\pi}{4}\sin\phi\right]. \tag{2.13}$$

Here we note that the $\cos\phi$ or the $\sin\phi$ modulation of the azimuthal distribution is proportional to the transverse polarisation of the spin–$\frac{1}{2}$ particles and also to the *analysing power* α. Thus, it is important that the production process yields a non-zero value of either η_1 or η_2. A non-zero η_2 indicates CP–violation or the presence final state interaction (absorptive parts). A non-zero η_1 can be obtained either with parity violation, which is present in the electro-weak sector of the SM or with appropriate initial beam polarisations. Further, we also need to know the analysing power α of the particle. For spin–$\frac{1}{2}$ particle, it is given in Eqs.(C.4) and (C.6). We see that the decay vertex has to be at least partially chiral, *i.e.* parity violating for $\alpha \neq 0$. That is, we need *effectively* chiral production and at least *partially* chiral decay for the fermions for their spin to be measured.
It is educative to realise that Eq.(2.12) can be cast into

$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\Omega_B} = \frac{1}{4\pi}\left[1+\alpha\frac{\vec{p}_B}{|\vec{p}_B|}.\vec{\eta}\right]. \quad \text{with} \quad \vec{\eta} = (\eta_1,\eta_2,\eta_3) \tag{2.14}$$

with $\vec{\eta}$ the polarisation vector. Performing a general rotation will leave $\vec{p}_B.\vec{\eta}$ unchanged. In the new frame, after rotation, we can define a new averaged azimuthal distribution as

$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\phi'} = \frac{1}{2\pi}\left[1+\frac{\alpha\eta_1'\pi}{4}\cos\phi' + \frac{\alpha\eta_2'\pi}{4}\sin\phi'\right]. \tag{2.15}$$

If the rotation is done along the η_2 direction (normal to the production plane), then $\eta_2' = \eta_2$ but η_1' will pick up a contribution from η_3, the average helicity. If $\eta_3 \gg \eta_1$ the

azimuthal distribution in this new frame is more conducive to a spin measurement, in the sense of catching the $\cos\phi$ dependence.

It is important to observe that the picture we have described so far in terms of azimuthal dependence through $\cos\phi$ and $\sin\phi$ (or higher for higher spins) may be very much impacted if cuts are applied to the cross section. If the cuts are ϕ-dependent, the azimuthal distributions may no longer have the simple form of Eq.(2.13) but would carry "spurious" dependence that would prevent the spin reconstruction as advocated here. Indeed, we could have a much more complicated dependence of the form

$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\phi} = \frac{1}{2\pi}\left[F_c(\phi) + \frac{\alpha\eta_1\pi}{4}G_c(\phi)\cos\phi + H_c(\phi)\frac{\alpha\eta_2\pi}{4}\sin\phi\right]. \tag{2.16}$$

unless only ϕ independent cuts are applied as suggested in Ref. [25].

2.2 Spin–1 particle

For the decay of spin–1 particle, $|1,l\rangle \to |s_1,l_1\rangle + |s_2,l_2\rangle$, the normalised decay density matrix is given by

$$\Gamma_1(l,l') = \begin{bmatrix} \frac{1+\delta+(1-3\delta)\cos^2\theta+2\alpha\cos\theta}{4} & \frac{\sin\theta(\alpha+(1-3\delta)\cos\theta)}{2\sqrt{2}}\ e^{i\phi} & (1-3\delta)\frac{(1-\cos^2\theta)}{4}\ e^{i2\phi} \\ \frac{\sin\theta(\alpha+(1-3\delta)\cos\theta)}{2\sqrt{2}}\ e^{-i\phi} & \delta+(1-3\delta)\frac{\sin^2\theta}{2} & \frac{\sin\theta(\alpha-(1-3\delta)\cos\theta)}{2\sqrt{2}}\ e^{i\phi} \\ (1-3\delta)\frac{(1-\cos^2\theta)}{4}\ e^{-i2\phi} & \frac{\sin\theta(\alpha-(1-3\delta)\cos\theta)}{2\sqrt{2}}\ e^{-i\phi} & \frac{1+\delta+(1-3\delta)\cos^2\theta-2\alpha\cos\theta}{4} \end{bmatrix}, \tag{2.17}$$

where,

$$\alpha = \frac{a_1^1 - a_{-1}^1}{a_1^1 + a_0^1 + a_{-1}^1} \quad , \quad \delta = \frac{a_0^1}{a_1^1 + a_0^1 + a_{-1}^1} \tag{2.18}$$

and

$$\begin{aligned} a_1^1 &= \left(\frac{3}{4\pi}\right)\sum_{l_1}|\mathcal{M}^1_{l_1,l_1-1}|^2 \qquad |l_1|\le s_1,\quad |l_1-1|\le s_2 \\ a_0^1 &= \left(\frac{3}{4\pi}\right)\sum_{l_1}|\mathcal{M}^1_{l_1,l_1}|^2 \qquad |l_1|\le \min(s_1,s_2) \\ a_{-1}^1 &= \left(\frac{3}{4\pi}\right)\sum_{l_1}|\mathcal{M}^1_{l_1,l_1+1}|^2 \qquad |l_1|\le s_1,\quad |l_1+1|\le s_2 \end{aligned} \tag{2.19}$$

The explicit calculation of the analysing power parameter α (the vector part) and δ (the rank–2 tensor) for a spin–1 particle decaying in two body final state is given in the Appendices C.3, C.4 and C.5. In particular $\delta = 0$ for decays into massless fermions assuming dimension–4operators. For the decay $W \to \bar{f}f'$ in the SM we have $\alpha = -1$ for massless f and f'.

The polarisation density matrix of spin–1 particle has two parts: the vector polarisation which we define here as $\vec{p} = (p_x, p_y, p_z)$ and is identical to that for a spin–$\frac{1}{2}$ $\vec{\eta}$ and the tensor polarisation described through a symmetric traceless rank–2 tensor T_{ij}, Tr T=0. The density matrix is parameterised as [33]

$$P_1(\lambda, \lambda') = \begin{bmatrix} \frac{1}{3} + \frac{p_z}{2} + \frac{T_{zz}}{\sqrt{6}} & \frac{p_x - ip_y}{2\sqrt{2}} + \frac{T_{xz} - iT_{yz}}{\sqrt{3}} & \frac{T_{xx} - T_{yy} - 2iT_{xy}}{\sqrt{6}} \\ \frac{p_x + ip_y}{2\sqrt{2}} + \frac{T_{xz} + iT_{yz}}{\sqrt{3}} & \frac{1}{3} - \frac{2T_{zz}}{\sqrt{6}} & \frac{p_x - ip_y}{2\sqrt{2}} - \frac{T_{xz} - iT_{yz}}{\sqrt{3}} \\ \frac{T_{xx} - T_{yy} + 2iT_{xy}}{\sqrt{6}} & \frac{p_x + ip_y}{2\sqrt{2}} - \frac{T_{xz} + iT_{yz}}{\sqrt{3}} & \frac{1}{3} - \frac{p_z}{2} + \frac{T_{zz}}{\sqrt{6}} \end{bmatrix}, \qquad (2.20)$$

Again using Eq.(2.8) we can write the angular distribution for spin–1 particle as

$$\begin{aligned} \frac{1}{\sigma_2} \frac{d\sigma_2}{d\Omega_B} = \frac{3}{8\pi} \Bigg[& \left(\frac{2}{3} - (1 - 3\delta) \, \frac{T_{zz}}{\sqrt{6}} \right) + \alpha \; p_z \cos\theta + \sqrt{\frac{3}{2}}(1 - 3\delta) \; T_{zz} \cos^2\theta \\ & + \left(\alpha \; p_x + 2\sqrt{\frac{2}{3}}(1 - 3\delta) \; T_{xz} \cos\theta \right) \sin\theta \; \cos\phi \\ & + \left(\alpha \; p_y + 2\sqrt{\frac{2}{3}}(1 - 3\delta) \; T_{yz} \cos\theta \right) \sin\theta \; \sin\phi \\ & + (1 - 3\delta) \left(\frac{T_{xx} - T_{yy}}{\sqrt{6}} \right) \sin^2\theta \cos(2\phi) \\ & + \sqrt{\frac{2}{3}}(1 - 3\delta) \; T_{xy} \; \sin^2\theta \; \sin(2\phi) \Bigg]. \end{aligned} \qquad (2.21)$$

The $\cos\theta$ averaged distribution is

$$\begin{aligned} \frac{1}{\sigma_2} \frac{d\sigma_2}{d\phi} = \frac{3}{4\pi} \Bigg[& \frac{2}{3} + \frac{\alpha p_x \pi}{4} \cos\phi + \frac{\alpha p_y \pi}{4} \sin\phi + \frac{2}{3}(1 - 3\delta) \left(\frac{T_{xx} - T_{yy}}{\sqrt{6}} \right) \cos(2\phi) \\ & + \frac{2}{3}(1 - 3\delta) \left(\sqrt{\frac{2}{3}} T_{xy} \right) \sin(2\phi) \Bigg]. \end{aligned} \qquad (2.22)$$

We note that the ϕ and 2ϕ modulation of the azimuthal distribution is proportional to the transverse polarisations, p_x and p_y, and the transverse components of the tensor polarisations, $T_{xx} - T_{yy}$ and T_{xy}. Again, in this frame all the cos modulations are CP–even and the sin modulations are CP–odd. To determine the spin we need $T_{xx} - T_{yy} \neq 0$ from the production part and $\delta \neq 1/3$ from the decay part in the CP–even production process. Since there is no symmetry that sets δ to be one-third, the dynamics in the decay part is not constrained. In other words, the decay mechanism does not require any parity violation.

As we have done for spin–1/2 it is instructive to rewrite Eq.(2.21) in terms of invariants under rotations. If one defines a rank–2 tensor out of the tensor product

of the unit vector describing the momentum of the decay product B, $\mathbb{P}_B = \vec{p}_B \otimes \vec{p}_B/|\vec{p}_B|^2$ [2], using the fact that T is traceless Eq.(2.21) writes in terms of (rotation) invariants as

$$\frac{1}{\sigma_2}\,\frac{d\sigma_2}{d\Omega_B} = \frac{1}{4\pi}\left[1+\alpha\,\frac{3}{2}\,\frac{\vec{p}_B}{|\vec{p}_B|}.\vec{p}+(1-3\delta)\sqrt{\frac{3}{2}}\,T.\mathbb{P}_B\right]. \tag{2.23}$$

We can then rewrite Eq.(2.23) in another frame, in particular one where we make a rotation around the y axis, transverse to the production plane. This will not mix the CP-odd and CP-even tensors but may make some ϕ asymmetries in the new frame larger.

2.3 spin–$\frac{3}{2}$ and spin–2

For spin–$\frac{3}{2}$ and spin–2 particles we give the decay density matrix in Appendix B. Since we need the coefficient of the highest harmonics to be non-zero for the spin to be determined, we note that for spin–$\frac{3}{2}$ we need parity violating interaction in the decay process, i.e., $\alpha_1 \neq 0$ and/or $\alpha_2 \neq 0$ whose combination defines the analysing power of highest rank. For spin–2 particles we need $A_4 \propto (a_2^2 - 4a_1^2 + 6a_0^2 - 4a_{-1}^2 + a_{-2}^2) \neq 0$. A_4 is the analysing power of rank–4, the highest rank for spin–2 to be non-vanishing (see Eq.(B.5)). This can be achieved without parity violating interactions in the decay process. We note that parity violating interactions are required in the decay of fermions for its spin to be measured along with its (transverse) polarisation being non-zero . For the bosons, on the other hand, we only need its transverse polarisation being non-zero either due to parity violation in the production process or due to polarisation of the initial beams.

3. The azimuthal distribution at event-generators/colliders

The azimuthal distributions Eqs.(2.13), (2.22) *etc.* are given in the rest frame of the decaying particle. To be able to measure the spin we need to construct the above mentioned azimuthal angle in terms of quantities defined in the lab frame of a collider experiment. Before considering other frames let us first define some asymmetries in the rest frame.

3.1 Asymmetries in the rest frame

We start with re-writing the rest frame azimuthal distribution in terms of some simple asymmetries that we define below. Let us first define

$$I_{2s}(\phi_1,\phi_2) = \int_{\phi_1}^{\phi_2} d\phi\,\frac{d\sigma_{2s}}{d\phi} \tag{3.1}$$

[2] $(\vec{p}_B \otimes \vec{p}_B)_{ij} = p_{B\,i}\,p_{B\,j}$. The scalar product is $T\mathbb{P}_B = \sum_{ij} T_{ij}\mathbb{P}_{B\,ij} = \mathrm{Tr}\;T\mathbb{P}_B$.

For $s = 1/2$ we define the following asymmetries and calculate them using Eq.(2.13):

$$\begin{aligned} A_1^1 &= \frac{I_1(-\pi/2,\pi/2) - I_1(\pi/2,3\pi/2)}{I_1(0,2\pi)} = \frac{\alpha\eta_1}{2} \\ B_1^1 &= \frac{I_1(0,\pi) - I_1(\pi,2\pi)}{I_1(0,2\pi)} = \frac{\alpha\eta_2}{2}. \end{aligned} \tag{3.2}$$

These asymmetries have been used in Ref. [32] as a probe of the polarisation of the top-quark. Eq.(2.13) can be re-written in terms of these asymmetries as

$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\phi} = \frac{1}{2\pi}\left[1 + \frac{\pi A_1^1}{2}\cos\phi + \frac{\pi B_1^1}{2}\sin\phi\right]. \tag{3.3}$$

Similarly for $s = 1$ we further define similar asymmetries and calculate them in terms of vector and tensor polarisations as follows,

$$\begin{aligned} A_2^1 &= \frac{I_2(-\pi/2,\pi/2) - I_2(\pi/2,3\pi/2)}{I_1(0,2\pi)} = \frac{3\alpha p_x}{4} \\ B_2^1 &= \frac{I_2(0,\pi) - I_2(\pi,2\pi)}{I_2(0,2\pi)} = \frac{3\alpha p_y}{4} \\ A_2^2 &= \frac{I_2(-\pi/4,\pi/4) - I_2(\pi/4,3\pi/4) + I_2(3\pi/4,5\pi/4) - I_2(5\pi/4,7\pi/4)}{I_2(0,2\pi)} \\ &= \frac{2}{\pi}\,(1-3\delta)\,\left(\frac{T_{xx}-T_{yy}}{\sqrt{6}}\right) \\ B_2^2 &= \frac{I_2(0,\pi/2) - I_2(\pi/2,\pi) + I_2(\pi,3\pi/2) - I_2(3\pi/2,2\pi)}{I_2(0,2\pi)} \\ &= \frac{2}{\pi}\,(1-3\delta)\left(\sqrt{\frac{2}{3}}T_{xy}\right). \end{aligned} \tag{3.4}$$

Thus Eq.(2.22) can be re-written as

$$\frac{1}{\sigma_2}\frac{d\sigma_2}{d\phi} = \frac{1}{2\pi}\left[1 + \frac{\pi A_2^1}{2}\cos\phi + \frac{\pi B_2^1}{2}\sin\phi + \frac{\pi A_2^2}{2}\cos(2\phi) + \frac{\pi B_2^2}{2}\sin(2\phi)\right]. \tag{3.5}$$

We see that the ϕ distribution in the rest frame of the decaying particle has very simple form in terms of the above mentioned asymmetries. It is clear that for higher spins we need to cut the 2π in more and more parts. The important observation to make is that the coefficient of the $\cos\phi$, A_1^1 for spin–1/2 and A_1^2 for spin–1 are determined exactly in the same way in terms of the asymmetries, this generalises also in the same to higher spin particles and similarly for other coefficients of $\cos j\phi$.

Next we write the asymmetries in terms of spin-momentum correlators. To this effect, we first define the following spin vectors in the helicity rest frame

$$s_x = (0,1,0,0), \quad s_y = (0,0,1,0), \quad s_z = (0,0,0,1) \tag{3.6}$$

Modulations	Asymmetries	Spin-momentum correlators $\mathcal{C}_j$
$\cos(\phi)$	A^1	$s_x.p_B$
$\sin(\phi)$	B^1	$s_y.p_B$
$\cos(2\phi)$	A^2	$(s_x.p_B)^2 - (s_y.p_B)^2$
$\sin(2\phi)$	B^2	$(s_x.p_B)(s_y.p_B)$
$\cos(3\phi)$	A^3	$(s_x.p_B)^3 - 3(s_x.p_B)(s_y.p_B)^2$
$\sin(3\phi)$	B^3	$3(s_x.p_B)^2(s_y.p_B) - (s_y.p_B)^3$
$\cos(4\phi)$	A^4	$(s_x.p_B)^4 - 6(s_x.p_B)^2(s_y.p_B)^2 + (s_y.p_B)^4$
$\sin(4\phi)$	B^4	$(s_x.p_B)^3(s_y.p_B) - (s_x.p_B)(s_y.p_B)^3$

Table 1: The table of asymmetries A^j and B^j corresponding to the $j\phi$ modulation of the azimuthal distribution and corresponding spin-momentum correlators $\mathcal{C}_j$. Here $s_{x,y}$ are the transverse spin directions of the decaying particle A, with s_x being in the production plane and p_B is the 4–momentum of the decay product B. The spin vectors $s_{x,y}$ are listed in Table 2 in different frames. For frame F we replace s_i with $\hat{s}_i$ in the above correlators.

which are orthogonal to the 4–momenta of the particle A, $p_A = (m_A, 0, 0, 0)$. These spin vectors satisfy the conditions $p.s_i = 0$ and $s_i.s_j = -\delta_{ij}$. The asymmetries for the spin–$\frac{1}{2}$ case can then be written as

$$A_1^1 = \frac{\sigma(s_x.p_B < 0) - \sigma(s_x.p_B > 0)}{\sigma(s_x.p_B < 0) + \sigma(s_x.p_B > 0)}, \qquad B_1^1 = \frac{\sigma(s_y.p_B < 0) - \sigma(s_y.p_B > 0)}{\sigma(s_y.p_B < 0) + \sigma(s_y.p_B > 0)}. \tag{3.7}$$

The asymmetries for the spin–1 case can be written as

$$A_2^1 = \frac{\sigma(s_x.p_B < 0) - \sigma(s_x.p_B > 0)}{\sigma(s_x.p_B < 0) + \sigma(s_x.p_B > 0)}, \qquad B_2^1 = \frac{\sigma(s_y.p_B < 0) - \sigma(s_y.p_B > 0)}{\sigma(s_y.p_B < 0) + \sigma(s_y.p_B > 0)},$$

$$A_2^2 = \frac{\sigma([s_x.p_B]^2 - [s_y.p_B]^2 > 0) - \sigma([s_x.p_B]^2 - [s_y.p_B]^2 < 0)}{\sigma([s_x.p_B]^2 - [s_y.p_B]^2 > 0) + \sigma([s_x.p_B]^2 - [s_y.p_B]^2 < 0)},$$

$$B_2^2 = \frac{\sigma([s_x.p_B][s_y.p_B] > 0) - \sigma([s_x.p_B][s_y.p_B] < 0)}{\sigma([s_x.p_B][s_y.p_B] > 0) + \sigma([s_x.p_B][s_y.p_B] < 0)} \quad . \tag{3.8}$$

It is clear from Eqs.(3.7) and (3.8) that the asymmetry corresponding to a given modulation of ϕ has identical expressions in terms of the spin-momentum correlator $s_x.p_B$ and $s_y.p_B$. What we mean is that by constructing specific functions with products of $s_i.p_B$ one reconstructs the set of $\cos m\phi$, $\sin m\phi$, see Table 1. Thus in a spin independent way we can write these asymmetries as

$$A^j \text{ or } B^j = (-1)^j \, \frac{\sigma(\mathcal{C}_j > 0) - \sigma(\mathcal{C}_j < 0)}{\sigma(\mathcal{C}_j > 0) + \sigma(\mathcal{C}_j < 0)} \quad , \tag{3.9}$$

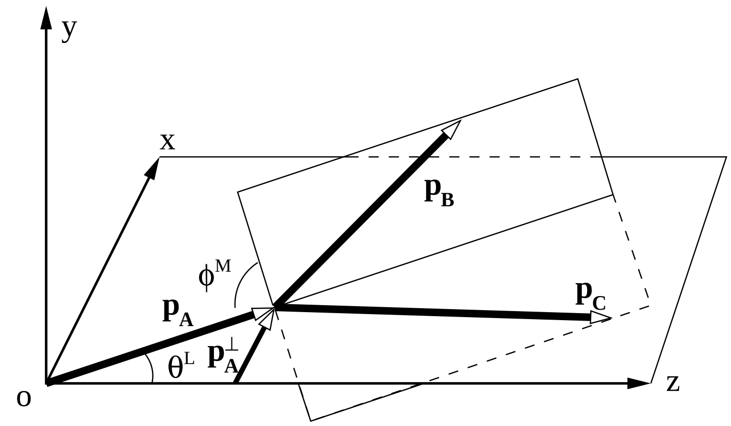

Figure 2: The momentum configuration in the *laboratory frame* L of the colliding beams is depicted. The angle between the production plane (the xz–plane) and the decay plane (spanned by p_B and p_C) is the azimuthal angle ϕ^M $(= \phi^M_B)$, which is the ordinary azimuthal angle in the frame M. This azimuthal angle has been studied in [23] and [24].

where the correlators $\mathcal{C}_j$s are listed in Table 1 for different modulations. Further we note that these expressions of asymmetries have a simple interpretation in terms of the polarisation parameters as long as $\vec{s_z}$, 3-vector, is parallel to the 3-momentum of the decaying particle in the frame of choice and $\vec{s_i}$ are orthogonal to each other. This defines a *helicity frame*. The lab frame, achieved by a boost along z-axis and then rotation around y-axis, also satisfies the properties of being a helicity frame. We note that the orthogonality of $\vec{s_i}$ is respected only if the boost is along one of $\vec{s_i}$ directions. In some other frame where these properties are not valid one needs to re-write these s_is as linear combinations of orthogonal s_i as we will see in the following sections. This will be necessary if the new frame is not reached through a boost made along the direction of motion of the particle.

3.2 The rotated frame M

The rotated frame is in fact the frame that is obtained from the rest frame by performing a pure Lorentz boost along the quantisation axis, the amount of boost is such that the energy of the particle whose spin we want to study is the same as the one measured in the laboratory frame. This is therefore a helicity frame in the sense that the quantisation axis has now been identified to lie along the momentum of the particle. The normalised 3–spin vectors $\vec{s_i}$ remain unchanged and therefore also the polarisation vectors $(\vec{\eta}, \vec{p})$ and other tensor polarisations. Hence the azimuthal asymmetries are the same as in the rest frame and have a one-to-one correspondance to the polarisation tensors defined in the rest frame. The appellation rotated frame comes from how this frame is picture in the laboratory frame. In fact this can be viewed as a simple rotation. In the laboratory frame L, defined in Table 2, the production plane of the particle A defines the xz–plane and the plane containing the

decay products with momenta p_B and p_C defines the decay plane. These two planes intersect along the momentum p_A of the decaying particle, see Fig. 2. Thus the angle between these two planes is the azimuthal angle of the decay product around the axis of spin quantisation (the momentum p_A), *i.e.* the ϕ we have mentioned in Eqs.(2.13) and (2.22) and which corresponds to exactly the azimuthal angle defined in the rest frame. In terms of the variables defined in the laboratory frame L, it is defined as [23, 24]

$$\phi = \cos^{-1}\left(\frac{(\hat{z}\times\vec{p}_A^{\,L}).(\vec{p}_C^{\,L}\times\vec{p}_B^{\,L})}{|\hat{z}\times\vec{p}_A^{\,L}|\;|\vec{p}_C^{\,L}\times\vec{p}_B^{\,L}|}\right) = \phi_B^M, \tag{3.10}$$

$$\text{where,}\quad \phi_B^M = \tan^{-1}\frac{s_y^M.p_B^M}{s_x^M.p_B^M} = \tan^{-1}\frac{s_y^L.p_B^L}{s_x^L.p_B^L}. \tag{3.11}$$

From Eq.(3.10) it appears that the reconstruction of this angle in the laboratory requires that one measures the momenta of *all* the decay products. This may be achieved even if C, say, is invisible provided one has enough control and constraints on the initial state so that the momentum of the decaying particle A is known, like for instance in e^+e^- annihilation where the beam energy is fixed.

Another way to get the azimuthal angle ϕ_M it to re-construct the scattering angle θ_A^L in the lab frame and rotate the event about the y-axis by that angle to bring the momentum p_A in the direction of the z-axis. The azimuthal angle of the decay product, ϕ_B^M is same as ϕ mentioned above in the lab frame. We dub this frame the *rotated frame* M (obtained by rotating the laboratory frame) and the momenta in this frame as compared to that in the laboratory frame is given in the Table 2. Using the form of the momenta p_B and s_i in frames R(rest frame) and M, one can see that $\phi = \phi_B^R = \phi_B^M$. Thus the distribution in angle ϕ, defined in Eq.(3.10), is the same azimuthal distribution as in the rest frame with same amplitudes for the different harmonics.

This azimuthal angle has been first studied in Ref. [23] to demonstrate the simple $\cos(j\phi)$ modulations of the azimuthal distributions and has been used to examine the spin of Z and W bosons at Tevatron and LEP-II, respectively, in Ref. [24]. Here we provide a theoretical understanding of the amplitude of these $\cos(j\phi)$ modulations in terms of transverse polarisations of the particle under consideration and its analysing power α *etc.*. In the event when the transverse polarisation is negligibly small or zero, this frame will not give any modulation in the azimuthal distribution as shown in section 4 for top pair production in e^+e^-. To address this potential issue we construct another frame which will give us an independent estimate on the modulations of the ϕ distribution and hence the spin of the particle. A hint on how to achieve this has been illustrated in section 2.1 and section 2.2 for the spin–1/2 and spin–1 when a simple rotation mixed the longitudinal polarisation and the transverse polarisation in the production plane, leaving the polarisation transverse to the production plane

Rest frame := R	Lab frame := L
$s_x^R = (0,1,0,0)$ $s_y^R = (0,0,1,0)$ $s_z^R = (0,0,0,1)$ $p_A^R = (m_A,0,0,0)$ $p_B^R = E_B^R \begin{pmatrix} 1 \\ \beta_B^R \sin\theta_B^R \cos\phi_B^R \\ \beta_B^R \sin\theta_B^R \sin\phi_B^R \\ \beta_B^R \cos\theta_B^R \end{pmatrix}$	$s_x^L = (0,\cos\theta_A^L,0,-\sin\theta_A^L)$ $s_y^L = (0,0,1,0)$ $s_z^L = (\beta_A^L,\sin\theta_A^L,0,\cos\theta_A^L)\gamma_A^L$ $p_A^L = E_A^L(1,\beta_A^L\sin\theta_A^L,0,\beta_A^L\cos\theta_A^L)$ $p_B^L = E_B^L \begin{pmatrix} 1 \\ \beta_B^L \sin\theta_B^L \cos\phi_B^L \\ \beta_B^L \sin\theta_B^L \sin\phi_B^L \\ \beta_B^L \cos\theta_B^L \end{pmatrix}$
Rotated frame := M	Boosted frame := F
$s_x^M = (0,1,0,0)$ $s_y^M = (0,0,1,0)$ $s_z^M = (\beta_A^L,0,0,1)\gamma_A^L$ $p_A^M = E_A^L(1,0,0,\beta_A^L)$ $p_B^M = E_B^L \begin{pmatrix} 1 \\ \beta_B^L \sin\theta_B^M \cos\phi_B^M \\ \beta_B^L \sin\theta_B^M \sin\phi_B^M \\ \beta_B^L \cos\theta_B^M \end{pmatrix}$	$\left.\begin{array}{l} \hat{s}_x^F = (0,1,0,0) \\ \hat{s}_y^F = (0,0,1,0) \\ \hat{s}_z^F = (\beta_A^F,0,0,1)\gamma_A^F \end{array}\right\}$ not the result of boost $L \to F$, see text. $p_A^F = E_A^F(1,0,0,\beta_A^F)$ $p_B^F = E_B^F \begin{pmatrix} 1 \\ \beta_B^F \sin\theta_B^F \cos\phi_B^F \\ \beta_B^F \sin\theta_B^F \sin\phi_B^F \\ \beta_B^F \cos\theta_B^F \end{pmatrix}$

Table 2: Momentum p_A, p_B and the spin directions s_i in various frames. The transformation $R \to M$ is a boost along z-axis $\Lambda_z(\beta_A^L)$, $M \to L$ is a rotation $R(\theta_A^L)$ and p_A^F & p_B^F are obtained by a boost along x-axis $\Lambda_x(-\beta_A^L \sin\theta_A^L)$ from frame L. Note that the spin vectors $\hat{s}_i^F$ in frame F are not related to s_i^L through boost but constructed such that they represent the helicity basis. The expressions for $s_i^F = \Lambda_x(-\beta_A^L \sin\theta_A^L)s_i^L$ which are the result of the boost are given in Eq. 3.12. The azimuthal angle of interest is $\phi = \tan^{-1}(s_y.p_B/s_x.p_B)$ in each frame (with s_i replaced by $\hat{s}_i$ in frame F).

unchanged. The next section will show how this can be achieved in general and how we can construct the spin-momentum correlators in this case.

3.3 The boosted frame F

The idea behind the boosted frame F is to induce a non zero azimuthal asymmetry even in the event that transverse tensor polarisations are very small or vanishing by making the longitudinal components, assuming it is non zero, contribute. We will show how this can be achieved especially how we can construct the correlators from combinations of variables measured in the laboratory frame. It should be added

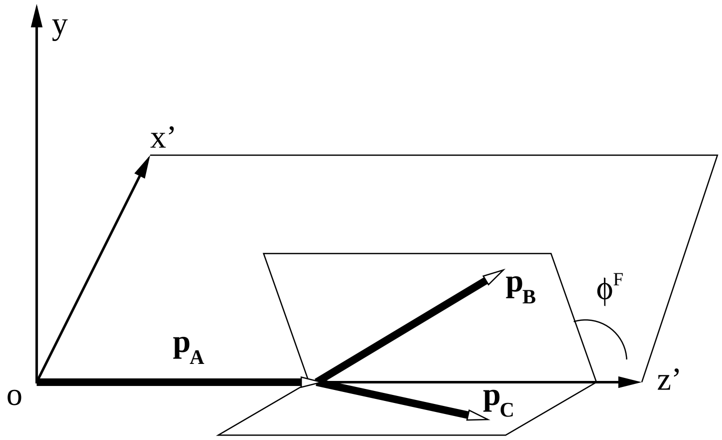

Figure 3: The momentum configuration in the transversely *boosted frame* F is depicted. The azimuthal angle of p_B is the azimuthal angle ϕ^F $(= \phi_B^F)$. This frame is obtained from frame L by boosting along negative x-axis such that the transverse momentum $p_A^\perp$ becomes zero.

that both the rest frame and the M frame are helicity frames. In the new frame and in order to arrive at the mixing between the longitudinal and the transverse polarisations we need to perform a transformation that will move the longitudinal spin (quantisation axis) away from the momentum of the particle. Yet, we still need to reconstruct a helicity basis in order to construct the helicity density matrix. To achieve the *misalignment*, we observe that a Lorentz boost in a direction other than the direction of the particle momentum will mix the helicity states. In our case one way to achieve this is to carry a boost from the laboratory frame along the negative x-axis with velocity $\beta_A^L \sin\theta_A^L$, thus reaching the *boosted frame* F. The momentum p_A^F of A is then pointing along the z-axis, see Table 2. This looks as if we have slowed down the particle, however contrary to frame M where the momentum is also pointing in the z direction, we can check that none of the transformed spin vectors $s_i^F = \Lambda_x(-\beta_A^L \sin\theta_A^L)\ s_i^L$ has its three momentum lying on the momentum of the particle. In fact, it can be shown on general ground that if initially the spin axis is parallel to the particle momentum, in the new frame these two axes will move away by an angle ω, the Wick angle [33], if the boost is not performed along the particle momentum. Indeed we verify that

$$\begin{aligned}
s_x^F &= \Lambda_x(-\beta_A^L \sin\theta_A^L)\ s_x^L = \cos\omega\ \hat{s}_x^F - \sin\omega\ \hat{s}_z^F \\
s_y^F &= \hat{s}_y^F = s_y^L \\
s_z^F &= \Lambda_x(-\beta_A^L \sin\theta_A^L)\ s_z^L = \sin\omega\ \hat{s}_x^F + \cos\omega\ \hat{s}_z^F \quad \text{with}
\end{aligned}$$
$$\cos\omega = \frac{\cos\theta_A^L}{\sqrt{1-(\beta_A^L \sin\theta_A^L)^2}}, \quad \sin\omega = \frac{\sin\theta_A^L}{\gamma_A^L\sqrt{1-(\beta_A^L \sin\theta_A^L)^2}} \qquad (3.12)$$

$\hat{s}_{x,y,z}^F$ are the helicity basis in the new frame F and are given explicitly in Table 2. Since the spin vectors $s_{x,z}^L$ in the laboratory frame are not parallel to the x-axis

(except for $\sin\theta_A^L = 0$ or 1), a boost along the x-axis modifies the orthogonality of the spatial component of s_i in the boosted frame. Also the spatial components of the boosted s_z, i.e. $\Lambda(-\beta_A^L \sin\theta_A^L) s_z^L$, is not parallel to the spatial component of p_A^F, owing to the Wick rotation of the spin basis. Thus the definition of the longitudinal or transverse polarisations in the frame F, which is not a helicity frame, is different from that in the helicity lab frame L. Since the asymmetries A^j and B^j in the frame F are defined with respect to $\hat{s}_i^F$, we can in principle have A^j non-zero even in the absence of any transverse polarisation in frame L. The helicity basis $\hat{s}_{x,y,z}^F$ is to be used to construct the spin-momentum correlators in frame F.
This rotation of the spin basis vectors leads to the transformation of the density matrix and the various polarisation parameters. In general, for a rotation defined through the, Euler, angle $\tilde{\theta}, \tilde{\phi}$ the density matrix transforms as [33]

$$\rho' = D(\tilde{\phi}, \tilde{\theta}, -\tilde{\phi})\ \rho^M\ D^\dagger(\tilde{\phi}, \tilde{\theta}, -\tilde{\phi}) \ .$$

In our case we have $\tilde{\theta} = \omega$ and $\tilde{\phi} = 0$(boost in the x direction), which leads to

$$\rho^F(\lambda, \lambda') = d^s_{\lambda l}(\omega)\ d^s_{\lambda' l'}(\theta_\omega)\ \rho^L(l, l') \ . \tag{3.13}$$

Thus the density matrix ρ^F does not receive any additional phase and the azimuthal $\sin(n\phi_B^F)$ dependence remains unaltered. The polarisation parameters, for the spin–1 for example as concerns the vector and the tensor polarization, transform as [33]

$$p_i^F = R_{ij}(\omega)\ p_j^L \ , \quad T_{i,j}^F = R_{ik}(\omega)\ R_{jk}(\omega)\ T_{kl}^L \tag{3.14}$$

as mentioned earlier in Sec. 2.1 and 2.2. Here, $i, j, k, l = \{x, y, z\}$, R_{ij} is the matrix, which for the boost we have performed, corresponds to a rotation about the y-axis in the cartesian coordinate. The superscript F or L stands for the quantities defined in frame F or frame L respectively. The asymmetries A^j and B^j in frame F have exactly the same expression as Eqs(3.2) and (3.4) with p_i and T_{ij} replaced by the ones defined in Eq.(3.14). Thus for a spin-1 particle we have $A^1 \propto p_x^F = R_{xj}(\omega) p_j^L$ and not simply related to the transverse polarisation p_x^L as we have in the (helicity) frames M or L. This shows again that one can have a non-zero A^1 even in the absence of transverse polarisation in the frames M or L.
The azimuthal angle in this boosted frame is denoted by ϕ_B^F and shown in Fig.3. Since ϕ_B^F is the azimuthal angle around the new momentum p_A^F, it will have a simple $\cos(j\phi)$ and $\sin(j\phi)$ modulations in the distribution up to $j = 2s$. The asymmetries A^j and B^j in this frame are defined w.r.t. the spin directions $\hat{s}_i^F$ given in the Table 2. ϕ_B^F is expressed as

$$\cot\phi_B^F = \frac{\hat{s}_x^F . p_B^F}{\hat{s}_y^F . p_B^F} = \frac{\cos\omega\ s_x^L . p_B^L + \sin\omega\ s_z^L . p_B^L}{s_y^L . p_B^L} \tag{3.15}$$

$$= \cos\omega\ \cot\phi_B^M + \sin\omega\ \frac{s_z^L . p_B^L}{s_y^L . p_B^L}. \tag{3.16}$$

We see that ϕ_B^F is related to ϕ_B^M in a non-trivial way and thus the corresponding modulation need not have vanishing amplitudes even when this is the case in terms of ϕ_B^M distribution. Note that the asymmetries can be zero in both frames if either we have $P_A(\lambda,\lambda') \propto \delta_{\lambda,\lambda'}$, *i.e.* when the particle is completely unpolarised, or when the particle is spin–0. In all other cases, the two frames will lead to different values of the azimuthal asymmetries. Thus, we need to use both frames to confirm the spin of the particle.

3.4 Note on event reconstruction

The asymmetries A^j and B^j and the azimuthal angles ϕ^M and ϕ^F require complete reconstruction of the test particle's momentum in order to construct the corresponding spin vectors s_i^L and/or $\hat{s}_i^{L/F}$. The possibility of reconstruction depend both on the kind of collider and the number of missing particles in the process. For example, reconstruction of spin vectors is possible at colliders with *fixed* center of mass energy, like the ILC, for some selected processes where the number of missing particles is 2 or less. At hadronic colliders, having variable centre of mass energy at partonic level, such reconstructions can be achieved for processes with one or no missing particles. Most of the new physics models with a dark matter candidate have two missing particles in the production process of new particles at LHC. This makes the desired re-construction as outlined here unfortunately impossible at LHC in such processes. It is worth further investigating how this method could be combined with other methods or improved. To illustrate the method we therefore turn to an application for a collider such as the ILC.

4. Application to $e^+e^- \to t\bar{t} \to bW^+\ \bar{b}W^- \to bl^+\nu\ \bar{b}jj$

In this section we study top-quark pair production in e^+e^- in the semi-leptonic channel as a test bed for the spin measurement based on the exploitation of the azimuthal asymmetries in different frames outlined previously. We chose this particular process because it represents a decay chain where the intermediate W boson is on-shell. The charged lepton, $l^+ = e^+, \mu^+$, in the leptonic decay of the top (and the W^+) will play the role of our particle B in the previous section and used to construct the spin-momentum correlators. In this example all the momenta can be reconstructed and therefore the methods we have outlined can be applied readily. We do not take beamsstrahlung into account nor do we consider the issue of backgrounds that might force us to introduce cuts, which we want to avoid. However, we consider the effect of beam polarisation. The polarisations of the initial electron and position beams can be used to tune the polarisation of the produced heavy particles that can drastically affect the polarisation. We work at $\sqrt{s} = 500$GeV where the total cross section, including branching ratios, is 81fb for unpolarised e^+, e^-, This corresponds to a total number of 40500 events with a typical luminosity of 500 fb^{-1}. For each fit we make,

we will indicate what the minimum number of events is required for a 3σ discovery of a particular $\cos j\phi$ modulation that is a measure of the spin of the particle, we will see that this programme could be successfully carried at a linear collider with 500 fb^{-1} for this process.

For event generation we use the partonic level event generator `Pandora-2.3` [34] and generate $2\ 10^6$ events for different initial state polarisations. Event by event we need to calculate $s_x^M.p_B^M, s_y^M.p_B^M$ (frame M) and $\hat{s}_x^F.p_B^F, \hat{s}_y^F.p_B^F$ (frame F) which can be expressed in terms of energies and angles measured in the lab frame,

$$\begin{aligned}
s_x^M.p_B^M &= s_x^L.p_B^L = -E_B^L(\cos\theta_A^L \sin\theta_B^L \cos\phi_B^L - \sin\theta_A^L \cos\theta_B^L) \\
\hat{s}_x^F.p_B^F &= \frac{E_B^L(\beta_A^L \sin\theta_A^L - \sin\theta_B^L \cos\phi_B^L)}{\sqrt{1-(\beta_A^L \sin\theta_A^L)^2}} \\
s_y^M.p_B^M &= = \hat{s}_y^F.p_B^F = s_y^L.p_B^L = -E_B^L \sin\theta_B^L \sin\phi_B^L \ .
\end{aligned} \tag{4.1}$$

We then calculate, for $A = t, W$ all the 8 asymmetries corresponding to the correlators in Table 1, therefore testing whether a value for the top spin as high as $s = 2$ is possible. The azimuthal angles in frames M and F can be constructed using Eq.(4.1) along with Eqs.(3.11) and (3.15) for generating the distributions. The reconstructed azimuthal distributions are then fitted with a general function

$$F_n(\phi) = a_0 + \sum_{j=1}^{n} [a_j \cos(j\phi) + b_j \sin(j\phi)] \tag{4.2}$$

with $n = 4$. With $n = 4$, the only bias is that the particle has spin $s \leq 2$. We then compare the best fit coefficients with the asymmetries calculated. Since we work with the SM production and decay mechanisms for the t–quark, there is no CP violation in this process. The fitting procedure returns $b_j \approx 0$ and $B^j \approx 0$ in both frames M and F for all the initial state polarisations. This constitutes therefore a consistency check and confirms the absence of CP violation. In the following sections we will only talk about the CP even contributions coming from various $\cos(j\phi)$ modulations and ignore the discussion on $\sin(j\phi)$ modulations as they are zero.

4.1 Spin–$\frac{1}{2}$ case: t–quark

The top pair production at an e^+e^- collider proceeds through a photon and a Z–boson exchange in the s–channel. We will study the effect of the initial polarisation of the electron P_{e^-} and positron P_{e^+}. The partial chiral nature of the Z coupling leads to a finite top polarisation even for unpolarised initial state electron and positron beams. For t–quark decaying into a lepton through a W, the analysing power of the top is $\alpha = 1$.

We start our analysis with unpolarised beams and the polarisation of top for this case is given as ($P_{e^\mp}$ is the polarisation of $e^\mp$):

$$(P_{e^-}, P_{e^+}) = (0.00, 0.00) : \eta_1 = +0.222, \quad \eta_2 = 0.000, \quad \eta_3 = -0.127 \ .$$

(P_{e^-}, P_{e^+})	Quantities	Frame M	Frame F	Reference
$(+0.00, +0.00)$	A^1	$+0.111$	-0.035	—
	a_1/a_0	$+0.175$	-0.055	
$(+0.80, -0.60)$	A^1	-0.253	$+0.149$	Fig. 4
	a_1/a_0	-0.397	$+0.234$	
$(+0.792, +0.60)$	A^1	≈ 0	$+0.021$	Fig. 5
	a_1/a_0	≈ 0	$+0.033$	

Table 3: Values for the fitted asymmetry A^1 and the fit parameter a_1/a_0, see Eq. 4.2, for the lepton distribution from t-decay for different initial state polarisations P_{e^-}, P_{e^+} for the electron and the positron in frames M and F. We have the relation $a_1/a_0 = \pi A^1/2$ which is observed numerically within tolerance ($\pm 10^{-3}$). The other $a_j/a_0, j \neq 1$ are zero within the tolerance. Recall that we generate $2\ 10^6$ events.

This corresponds to the asymmetry $A^1 = \eta_1/2 = 0.111$ and the amplitude of $\cos\phi$ to be $\pi A^1/2 = 0.175$. This is confirmed by the fit in frame M, see Table 3. We would need[3] $N_M \approx 730$ events to measure it with 3σ significance in frame M. In frame F the asymmetry in this case is $A^1 = -0.035$ and requires $N_F \approx 7350$ events for it to be measured with 3σ significance. Thus, one needs at least $\max(N_M, N_F) = 7350$ events to confirm the spin of t–quark to be at least $\frac{1}{2}$ with unpolarised beams. In this case where the beams are not polarised, the asymmetries are smaller in frame F however the analysis in this frame does confirm that no new modulation has been missed, and thus reconfirms the spin–1/2 nature of the top.

In order to improve the sensitivity, one might consider the case of polarised e^+e^- beams to produce top quarks with larger polarisation. For example,

$$(P_{e^-}, P_{e^+}) = (+0.80, -0.60): \quad \eta_1 = -0.505, \quad \eta_2 = 0.000, \quad \eta_3 = +0.554\ ,$$

which corresponds to much larger polarisation and hence a larger asymmetry $A^1 = -0.253$ in frame M. This requires only $N_M \approx 140$ events to measure A^1 with 3σ significance. The azimuthal distribution for this beam polarisation is shown in Fig. 4 in both the frames M and F. In frame F however the asymmetry A^1 is smaller, see Table 3, hence we need a larger number of events, $N_F \approx 410$, to measure it with 3σ significance. Thus, one needs $\max(N_M, N_F) = 410$ events to confirm the spin of t–quark to be at least $\frac{1}{2}$ with this choice of beam polarisation, which is a large improvement over the unpolarised case. To rule out higher asymmetries with a higher degree of significance one still needs a larger number of events than this.

[3]Number of events required: $N = f^2/(A^j)^2$, where f is the degree of statistical significance. Numbers with $f = 3$, for 3σ significance, are quoted.

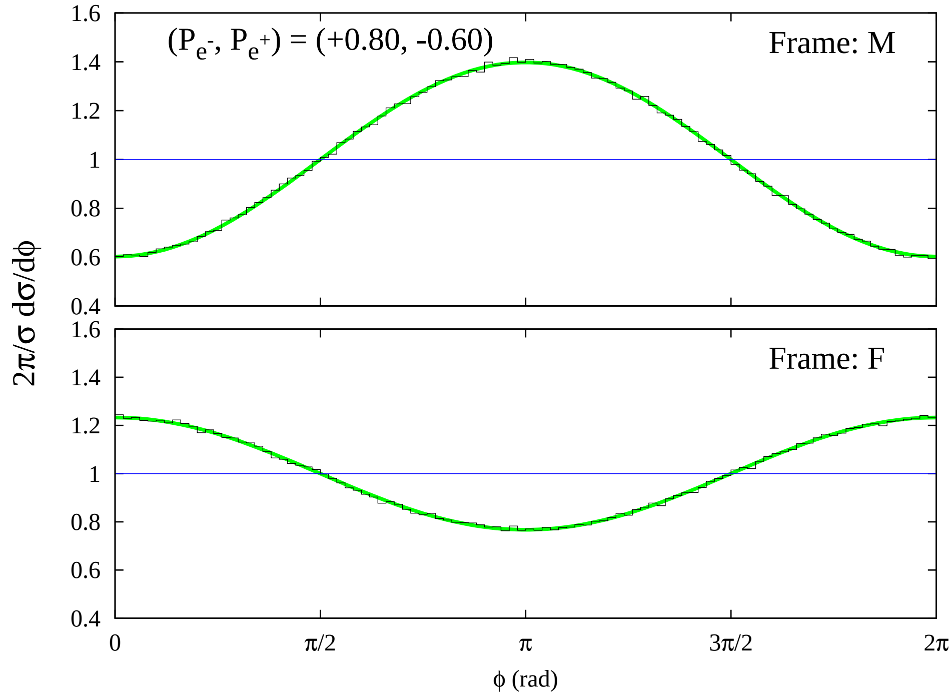

Figure 4: The azimuthal distribution of lepton from decay of t–quark is plotted for $(P_{e^-}, P_{e^+}) = (+0.80, -0.60)$ in frame M (top) and in frame F (below) using 2×10^6 events at partonic level (histogram). The best fit (green/grey line) to $F_4(\phi)$ leads to the coefficient of the $\cos\phi$ modulation to be non-zero (given in Table 3) and all other modulations are absent in both the frames indicating the spin of t–quark to be $\frac{1}{2}$.

Next we discuss the case when the transverse polarisation of t–quark, η_1, is zero. We arrange this by tuning the beam polarisations to appropriate values. This leads to $A^1 \approx 0$, and hence in frame M the fit gives $a_1/a_0 \approx 0$, and a flat distribution as shown in Fig. 5. The top polarisations in this case are given as

$$(P_{e^-}, P_{e^+}) = (+0.792, +0.60): \quad \eta_1 = 0.000, \quad \eta_2 = 0.000, \quad \eta_3 = +0.080 \ .$$

We note that the longitudinal polarisation of t–quark, η_3, though small is not zero and hence in frame F this leads to a non-zero value of the asymmetry A^1 and the $\cos\phi$ modulation as seen in Fig. 5. In this case $N_F \approx 2\ 10^4$ events are required to measure this asymmetry at 3σ significance. This example re-imposes the need for a second frame F in association with the helicity frame M to measure and re-confirm the spin of a particle.

4.2 Spin–1 case: W–boson

The W boson analysis is a much better advocate for the need of frame F, beside frame M. In the process under consideration, the W–bosons are produced (almost) on-mass-shell as a decay product of t–quark. Since the coupling of W–boson is chiral,

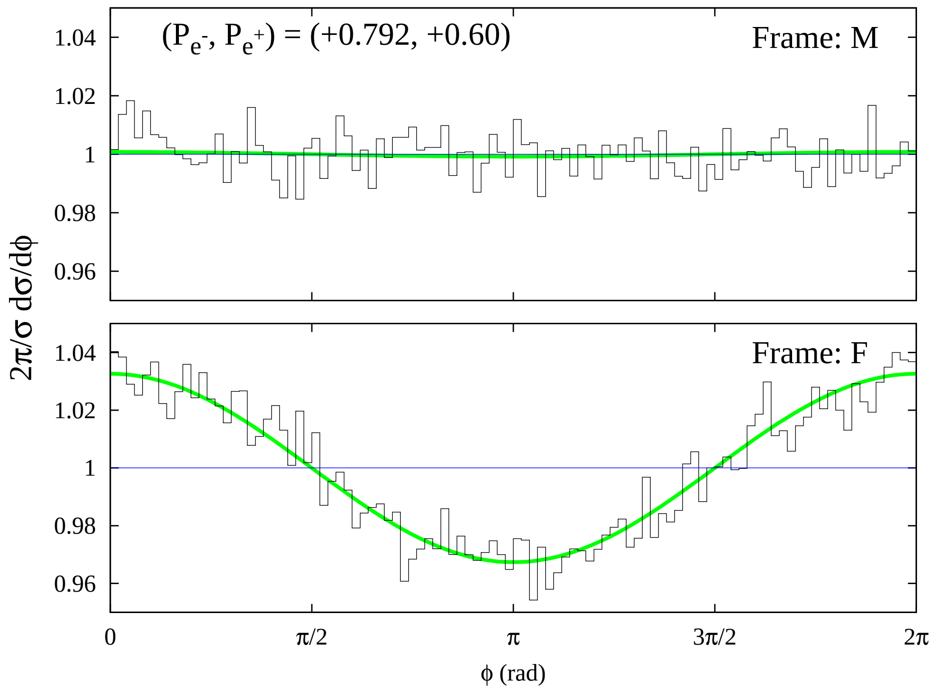

Figure 5: The azimuthal distribution same as in Fig. 4 for $(P_{e^-}, P_{e^+}) = (+0.792, +0.60)$. This is special case when the distribution is flat in frame M and frame F is needed for spin determination.

they are produced with high polarisation even in the decay of unpolarised top quarks. For the same set of events as used for the case of top quarks, the vector polarisations of the W–boson are given by

$$(P_{e^-}, P_{e^+}) = (+0.80, -0.60): \quad p_x = +0.355, \quad p_y = 0.000, \quad p_z = 0.000\ .$$

From Eq.(3.4) we know that the coefficient of the $\cos\phi$ modulation is proportional to p_x and non-zero in this case. The coefficient of $\cos 2\phi$ modulation is proportional to tensor polarisation $(T_{xx} - T_{yy})$, which happens to be zero[4] for this process in the helicity frame M. This leads to only $\cos\phi$ modulation of the azimuthal distribution as seen in Fig. 6 for frame M. Using the helicity amplitudes given in Appendix C.1, one can write the production density matrix for W–boson, which is produced in the decay of t–quark, and we easily see that $\rho_W(+1, -1) = \rho_W(-1, +1) = 0$ in the helicity frame M due to angular momentum conservation. Here, a higher spin (spin–1) particle is produced in the decay process of lower spin (spin–$\frac{1}{2}$), thus it can not span all its helicity states for fixed helicities of other particles and hence leads to $\rho_W(\pm 1, \mp 1) = 0$. This is proven for the general case in Appendix D in the helicity

[4]We note that the asymmetry A^2 is zero in the helicity frame M for on-shell W bosons, but numerically we find it to be small but non-zero as the decay width of W is not very small and there is a non-negligible contribution from off-shell Ws.

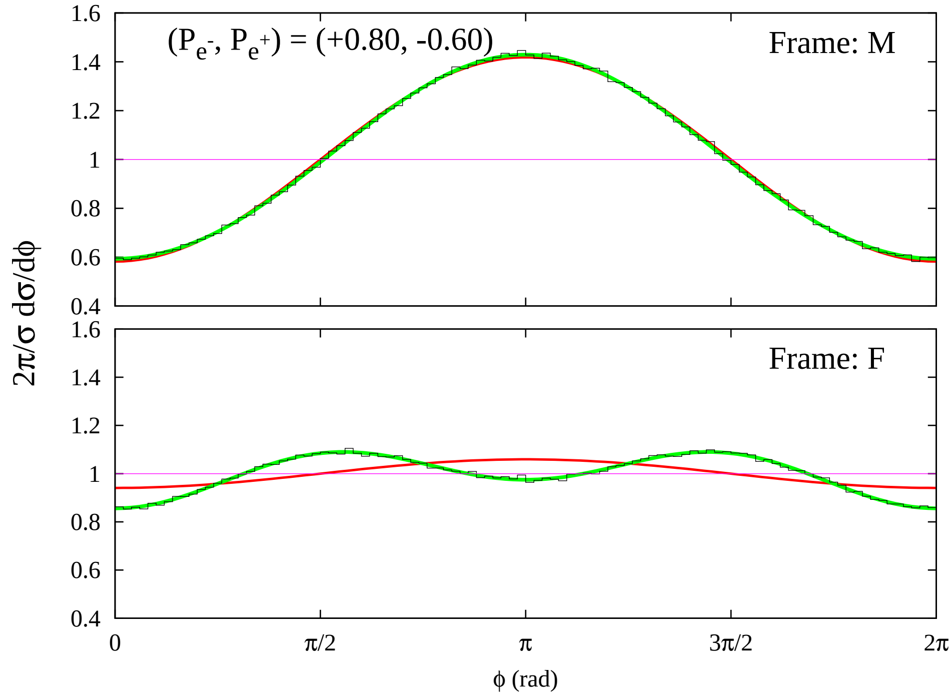

Figure 6: The azimuthal distribution of lepton from decay of W–boson is plotted for $(P_{e^-}, P_{e^+}) = (+0.80, -0.60)$ in frame M (top) and in frame F (below) using 2×10^6 events at partonic level (histogram). The best fit (green/grey line) to $F_4(\phi)$ leads to the coefficient of the $\cos\phi$ and $\cos 2\phi$ modulation to be non-zero in frame F (given in Table 4) and all other modulations are absent in both the frames indicating the spin of W–boson to be 1. The red (dark grey) line show only the $\cos\phi$ modulation of the distribution.

frame. However, in the boosted frame F the asymmetry A^2 measures $T^F_{xx} - T^F_{yy}$, which is non-zero in general, see Eq.(3.14). In frame F we find $A^2 = -0.054$ which leads to a $\cos 2\phi$ modulation of the azimuthal angle in this frame, see Fig. 6. Here $N_F = 3100$ events will be required to measure A^2 with 3σ significance. Further, all the higher A^js ($A^{j>2}$) are found to be zero in both frames proving that the particle under consideration to be spin–1 and its production process to be CP-conserving. The asymmetries and fit parameters are listed in Table 4 for both the frames.

Next we look at a case where the azimuthal distribution in the helicity frame M is flat which would wrongly suggest that the particle is a scalar. The various vector polarisations are given as

$$(P_{e^-}, P_{e^+}) = (+0.75, +0.60): \quad p_x = +0.000, \quad p_y = 0.000, \quad p_z = 0.193 \; .$$

In the helicity frame M, the asymmetry A^2 is zero due to the angular momentum conservation and A^1 is zero because it is proportional to p_x, which is zero for the chosen initial state beam polarisations. The W–boson appears to be spin–0 in this frame M with this particular beam polarisations. The asymmetries A^j and the fit

(P_{e^-}, P_{e^+})	Quantities	Frame M	Frame F	Reference
$(+0.80, -0.60)$	A^1	-0.266	-0.038	Fig. 6
	A^2	≈ 0	-0.054	
	a_1/a_0	-0.418	-0.059	
	a_2/a_0	≈ 0	-0.086	
$(+0.75, +0.60)$	A^1	≈ 0	-0.093	Fig. 7
	A^2	≈ 0	-0.026	
	a_1/a_0	≈ 0	-0.147	
	a_2/a_0	≈ 0	-0.041	

Table 4: The table of asymmetries A^1 & A^2 and the fit parameter a_1/a_0 & a_2/a_0 for lepton's distribution from W–boson decay for different initial state polarisations in frames M and F. We have the relation $a_i/a_0 = \pi A^i/2$, which is also observed numerically within tolerance ($\pm 10^{-3}$).

parameters a_j/a_0 are listed in Table 4 for this case and the corresponding azimuthal distributions are plotted in Fig. 7. Changing over to frame F leads to non-zero values of both A^1 and A^2, see Table 4, and the corresponding azimuthal distribution visibly has the $\cos 2\phi$ modulation, Fig. 7. In this case $N_F \approx 1.3 \times 10^4$ events are required to measure A^2 with 3σ significance. This is the best example of a case where one needs a frame other than the helicity frame to confirm the spin of the particle, which is polarised with $p_z \neq 0$ and $T_{zz} \neq 0$. We, however, note that this process is not the best process to study the spin of W-boson. For this purpose one should look at the pair production process $e^+e^- \to W^+W^-$ as discussed in Ref. [24].

Thus we conclude that one needs two different reference frames to measure and re-confirm the spin of a particle using the same set of events. Further, if the event set includes a cascade decay, one can construct the asymmetries for different particles using the spin vector s_i for different particles and using the momentum p_B of different final state particles. For example, in the above case, we could have used the momentum of the b–quark to construct the asymmetries in place of the leptons. In the events with hadronic decay of Ws, one could use either of the jets to construct the correlators and hence the asymmetries. Thus, using different final state particles, we can find a larger set of asymmetries to re-confirm the spin of a particle, however we can not improve the significance of the measurement by combining different correlators for the same set of events. A larger event sample is necessary to improve the statistical significance of the measurements.

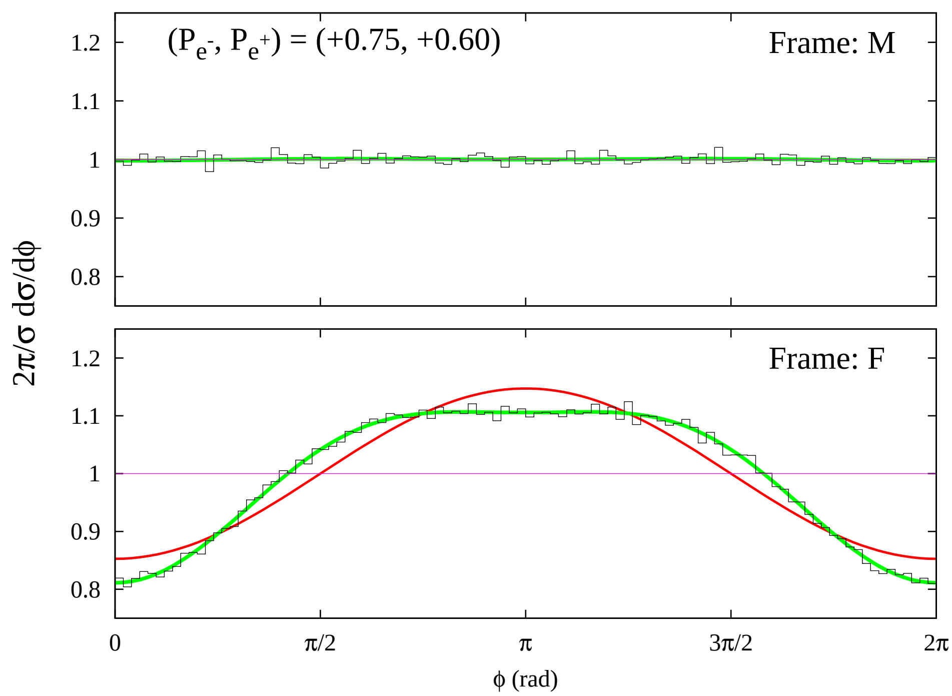

Figure 7: The azimuthal distribution same as in Fig. 6 for $(P_{e^-}, P_{e^+}) = (+0.75, +0.60)$. This is special case where frame M has flat distribution and frame F is needed for the spin determination.

5. Discussions and Conclusions

In this paper, we constructed observables to measure the spin of a heavy unstable particle produced at a collider by harvesting the spin dependence of the azimuthal distribution through the quantum interference between the different helicity states, *i.e* the non diagonal elements of the helicity density matrix. The aim is to construct observables that are sensitive to the highest rank–$2s$ tensor polarisation of a particle with spin–s that lead to a $\cos 2s\phi$ modulation of the azimuthal distribution. Such a method has been known for a long time and we have provided an analytical understanding of it. In particular, the novelty of our approach is the construction of two reference frames where in one of them the spin basis is subjected to a Wick rotation. The latter mixes the longitudinal polarisation and the transverse polarisation in the production plane and therefore the spin modulation in the azimuthal angle is sensitive to this mixture whereas in the standard approach the longitudinal polarisation is integrated away and does not contribute to the usual azimuthal asymmetries. The construction of two frames allows within the same experiment and with the same event sample to cross check the spin measurement based on azimuthal asymmetries. In some cases this can be crucial since the usual transverse polarisation tensor/vector can be zero either accidentally or for a dynamical reason and therefore would lead to a wrong conclusion. This can be rescued in the second frame provided the longi-

tudinal polarisation is not zero as well. We have shown examples in the decay of the top and the W where this occurs with SM production and decay mechanisms. In the appendices we consider more general couplings and decays than those that describe the SM particles. This helps in obtaining a set of conditions on the production and decay mechanisms for some of the asymmetries to be non-zero and hence the spin to be measured. One drawback of the method however, as outlined in the present paper, is that it requires complete reconstruction of the test particle's momentum which is necessary to build up the needed spin vectors. If there are too many invisible particles this might not be possible especially in a machine like the LHC where the partonic centre of mass energy is not fixed. The same drawback also affects other methods of spin reconstruction. We feel however that is worth investigating how the method we have described can be exploited in combination with other methods or by making some mild assumption on the spectrum of the event or the underlying physics.

Acknowledgments

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A. Rotation matrices $d^j_{m,n}(\theta)$

The general form of the d function is given in Eq.(1.2). For completeness and although these can be easily found in many textbooks, we explicitly write the d function up to spin–2. To avoid clutter we take as short-hand notation $c = \cos(\theta/2)$ and $s = \sin(\theta/2)$ then all the $d^j_{m,n}$ useful to our study are given below.

- $j = 0 : \; d^0_{0,0} = 1$

- $j = \frac{1}{2} : \; d^{\frac{1}{2}}_{m,n} = \begin{bmatrix} c & -s \\ s & c \end{bmatrix}$

- $j = 1 : \; d^1_{m,n} = \begin{bmatrix} c^2 & -\sqrt{2}cs & s^2 \\ \sqrt{2}cs & 2c^2-1 & -\sqrt{2}cs \\ s^2 & \sqrt{2}cs & c^2 \end{bmatrix}$

- $j = \frac{3}{2}$: $d^{\frac{3}{2}}_{m,n} = \begin{bmatrix} c^3 & -\sqrt{3}sc^2 & \sqrt{3}s^2c & -s^3 \\ \sqrt{3}sc^2 & c-3s^2c & s-3sc^2 & \sqrt{3}s^2c \\ \sqrt{3}s^2c & -s+3sc^2 & c-3s^2c & -\sqrt{3}sc^2 \\ s^3 & \sqrt{3}s^2c & \sqrt{3}sc^2 & c^3 \end{bmatrix}$
- $j = 2$: $d^2_{m,n} =$

$$\begin{bmatrix} c^4 & -2sc^3 & \sqrt{6}s^2c^2 & -2s^3c & s^4 \\ 2sc^3 & -3c^2+4c^4 & \sqrt{6}sc(s^2-c^2) & 3s^2-4s^4 & -2s^3c \\ \sqrt{6}s^2c^2 & -\sqrt{6}sc(s^2-c^2) & 1-6c^2+6c^4 & \sqrt{6}sc(s^2-c^2) & \sqrt{6}s^2c^2 \\ 2s^3c & 3s^2-4s^4 & -\sqrt{6}sc(s^2-c^2) & -3c^2+4c^4 & -2sc^3 \\ s^4 & 2s^3c & \sqrt{6}s^2c^2 & 2sc^3 & c^4 \end{bmatrix}$$

B. Decay density matrix for higher spin particle

As a short-hand notation we now define $C = \cos\theta$ and $S = \sin\theta$ which enter the expressions for the density matrices of higher spin particles, namely $s = 3/2$ and $s = 2$ briefly discussed in the main text. The corresponding normalised decay matrices are calculated from Eq. 2.7 and using the explicit expressions for the d matrices.

B.1 Spin-$\frac{3}{2}$ particle

For the decay $|\frac{3}{2}, l\rangle \to |s_1, l_1\rangle + |s_2, l_2\rangle$, the decay density matrix is given by

$$\Gamma^{\frac{3}{2}}(+\frac{3}{2},+\frac{3}{2}) = \frac{(1+2\gamma_1)+3(\alpha_1+\alpha_2)C+3(1-2\gamma_1)C^2+(\alpha_2-3\alpha_1)C^3}{8}$$
$$\Gamma^{\frac{3}{2}}(+\frac{3}{2},+\frac{1}{2}) = \frac{\sqrt{3}\ S\ [\ (\alpha_1+\alpha_2)+2(1-2\gamma_1)C+(\alpha_2-3\alpha_1)C^2\ ]}{8}\ e^{i\phi}$$
$$\Gamma^{\frac{3}{2}}(+\frac{3}{2},-\frac{1}{2}) = \frac{\sqrt{3}\ S^2\ [\ (1-2\gamma_1)+(\alpha_2-3\alpha_1)C\ ]}{8}\ e^{i2\phi}$$
$$\Gamma^{\frac{3}{2}}(+\frac{3}{2},-\frac{3}{2}) = \frac{(\alpha_2-3\alpha_1)S^3}{8}\ e^{i3\phi}$$
$$\Gamma^{\frac{3}{2}}(+\frac{1}{2},+\frac{3}{2}) = \frac{\sqrt{3}\ S\ [\ (\alpha_1+\alpha_2)+2(1-2\gamma_1)C+(\alpha_2-3\alpha_1)C^2\ ]}{8}\ e^{-i\phi}$$
$$\Gamma^{\frac{3}{2}}(+\frac{1}{2},+\frac{1}{2}) = \frac{(3-2\gamma_1)+(3\alpha_2-5\alpha_1)C-3(1-2\gamma_1)C^2-3(\alpha_2-3\alpha_1)C^3\ ]}{8}$$
$$\Gamma^{\frac{3}{2}}(+\frac{1}{2},-\frac{1}{2}) = \frac{S\ [\ (3\alpha_2-\alpha_1)-3(\alpha_2-3\alpha_1)C^2\ ]}{8}\ e^{i\phi}$$
$$\Gamma^{\frac{3}{2}}(+\frac{1}{2},-\frac{3}{2}) = \frac{\sqrt{3}\ S^2\ [\ (1-2\gamma_1)-(\alpha_2-3\alpha_1)C\ ]}{8}\ e^{i2\phi}$$
$$\Gamma^{\frac{3}{2}}(-\frac{1}{2},+\frac{3}{2}) = \frac{\sqrt{3}\ S^2\ [\ (1-2\gamma_1)+(\alpha_2-3\alpha_1)C\ ]}{8}\ e^{-i2\phi}$$

$$
\begin{aligned}
\Gamma^{\frac{3}{2}}(-\tfrac{1}{2},+\tfrac{1}{2}) &= \frac{S\ [\ (3\alpha_2-\alpha_1)-3(\alpha_2-3\alpha_1)C^2\ ]}{8}\ e^{-i\phi}\\
\Gamma^{\frac{3}{2}}(-\tfrac{1}{2},-\tfrac{1}{2}) &= \frac{(3-2\gamma_1)-(3\alpha_2-5\alpha_1)C-3(1-2\gamma_1)C^2+3(\alpha_2-3\alpha_1)C^3\ ]}{8}\\
\Gamma^{\frac{3}{2}}(-\tfrac{1}{2},-\tfrac{3}{2}) &= \frac{\sqrt{3}\ S\ [\ (\alpha_1+\alpha_2)-2(1-2\gamma_1)C+(\alpha_2-3\alpha_1)C^2\ ]}{8}\ e^{i\phi}\\
\Gamma^{\frac{3}{2}}(-\tfrac{3}{2},+\tfrac{3}{2}) &= \frac{(\alpha_2-3\alpha_1)S^3}{8}\ e^{-i3\phi}\\
\Gamma^{\frac{3}{2}}(-\tfrac{3}{2},+\tfrac{1}{2}) &= \frac{\sqrt{3}\ S^2\ [\ (1-2\gamma_1)-(\alpha_2-3\alpha_1)C\ ]}{8}\ e^{-i2\phi}\\
\Gamma^{\frac{3}{2}}(-\tfrac{3}{2},-\tfrac{1}{2}) &= \frac{\sqrt{3}\ S\ [\ (\alpha_1+\alpha_2)-2(1-2\gamma_1)C+(\alpha_2-3\alpha_1)C^2\ ]}{8}\ e^{-i\phi}\\
\Gamma^{\frac{3}{2}}(-\tfrac{3}{2},-\tfrac{3}{2}) &= \frac{(1+2\gamma_1)-3(\alpha_1+\alpha_2)C+3(1-2\gamma_1)C^2-(\alpha_2-3\alpha_1)C^3}{8}
\end{aligned}
\tag{B.1}
$$

where,

$$
\alpha_1 = \frac{a^{3/2}_{1/2}-a^{3/2}_{-1/2}}{\sum_l a^{3/2}_l},\qquad \alpha_2 = \frac{a^{3/2}_{3/2}-a^{3/2}_{-3/2}}{\sum_l a^{3/2}_l},\qquad \gamma_1 = \frac{a^{3/2}_{1/2}+a^{3/2}_{-1/2}}{\sum_l a^{3/2}_l} \tag{B.2}
$$

and

$$
\begin{aligned}
a^{3/2}_{3/2} &= \frac{1}{\pi}\sum_{l_1}|\mathcal{M}^{3/2}_{l_1,l_1-\frac{3}{2}}|^2 &\qquad |l_1|\le s_1,\quad |l_1-\tfrac{3}{2}|\le s_2\\
a^{3/2}_{1/2} &= \frac{1}{\pi}\sum_{l_1}|\mathcal{M}^{3/2}_{l_1,l_1-\frac{1}{2}}|^2 &\qquad |l_1|\le s_1,\quad |l_1-\tfrac{1}{2}|\le s_2\\
a^{3/2}_{-1/2} &= \frac{1}{\pi}\sum_{l_1}|\mathcal{M}^{3/2}_{l_1,l_1+\frac{1}{2}}|^2 &\qquad |l_1|\le s_1,\quad |l_1+\tfrac{1}{2}|\le s_2\\
a^{3/2}_{-3/2} &= \frac{1}{\pi}\sum_{l_1}|\mathcal{M}^{3/2}_{l_1,l_1+\frac{3}{2}}|^2 &\qquad |l_1|\le s_1,\quad |l_1+\tfrac{3}{2}|\le s_2
\end{aligned}
\tag{B.3}
$$

B.2 Spin-2 particle

For the decay $|2,l\rangle \to |s_1,l_1\rangle + |s_2,l_2\rangle$, the decay density matrix is given by

$$
\begin{aligned}
\Gamma^2(+2,+2) &= \left[\ A_0+4A_1C+6A_2C^2+4A_3C^3+A_4C^4\ \right]\\
\Gamma^2(+2,+1) &= 2\left[\ A_1+3A_2C+3A_3C^2+A_4C^3\ \right]\ S\ e^{i\phi}\\
\Gamma^2(+2,+0) &= \sqrt{6}\left[\ A_2+2A_3C+A_4C^2\ \right]\ S^2\ e^{i2\phi}\\
\Gamma^2(+2,-1) &= 2\left[\ A_3+A_4C\ \right]\ S^3\ e^{i3\phi}\\
\Gamma^2(+2,-2) &= A_4\ S^4\ e^{i4\phi}
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^2(+1,+2) &= 2\left[\ A_1 + 3A_2C + 3A_3C^2 + A_4C^3\ \right]\ S\ e^{-i\phi} \\
\Gamma^2(+1,+1) &= 4\left[\ 1 + 2(A_1 - 3\beta)C - 3A_2C^2 - 2A_3C^3 - A_4C^4\ \right] \\
\Gamma^2(+1,+0) &= 2\sqrt{6}\left[\ 2(\beta + A_2C) + S^2\ (A_3 + A_4C)\ \right]\ S\ e^{i\phi} \\
\Gamma^2(+1,-1) &= 4\left[\ 3A_2 + A_4S^2\ \right]\ S^2\ e^{i2\phi} \\
\Gamma^2(+1,-2) &= 2\left[\ A_3 - A_4C\ \right]\ S^3\ e^{i3\phi} \\
\Gamma^2(+0,+2) &= \sqrt{6}\left[\ A_2 + 2A_3C + A_4C^2\ \right]\ S^2\ e^{-i2\phi} \\
\Gamma^2(+0,+1) &= 2\sqrt{6}\left[\ 2(\beta + A_2C) + S^2\ (A_3 + A_4C)\ \right]\ S\ e^{-i\phi} \\
\Gamma^2(+0,+0) &= 4\left[\ 4\delta + 3A_2S^2 + 3A_4S^4\ \right] \\
\Gamma^2(+0,-1) &= 2\sqrt{6}\left[\ 2(\beta - A_2C) + S^2\ (A_3 - A_4C)\ \right]\ S\ e^{i\phi} \\
\Gamma^2(+0,-2) &= \sqrt{6}\left[\ A_2 - 2A_3C + A_4C^2\ \right]\ S^2\ e^{i2\phi} \\
\Gamma^2(-1,+2) &= 2\left[\ A_3 + A_4C\ \right]\ S^3\ e^{-i3\phi} \\
\Gamma^2(-1,+1) &= 4\left[\ 3A_2 + A_4S^2\ \right]\ S^2\ e^{-i2\phi} \\
\Gamma^2(-1,+0) &= 2\sqrt{6}\left[\ 2(\beta - A_2C) + S^2\ (A_3 - A_4C)\ \right]\ S\ e^{-i\phi} \\
\Gamma^2(-1,-1) &= 4\left[\ 1 - 2(A_1 - 3\beta)C - 3A_2C^2 + 2A_3C^3 - A_4C^4\ \right] \\
\Gamma^2(-1,-2) &= 2\left[\ A_1 - 3A_2C + 3A_3C^2 - A_4C^3\ \right]\ S\ e^{i\phi} \\
\Gamma^2(-2,+2) &= A_4\ S^4\ e^{-i4\phi} \\
\Gamma^2(-2,+1) &= 2\left[\ A_3 - A_4C\ \right]\ S^3\ e^{-i3\phi} \\
\Gamma^2(-2,+0) &= \sqrt{6}\left[\ A_2 - 2A_3C + A_4C^2\ \right]\ S^2\ e^{-i2\phi} \\
\Gamma^2(-2,-1) &= 2\left[\ A_1 - 3A_2C + 3A_3C^2 - A_4C^3\ \right]\ S\ e^{-i\phi} \\
\Gamma^2(-2,-2) &= \left[\ A_0 - 4A_1C + 6A_2C^2 - 4A_3C^3 + A_4C^4\ \right]
\end{aligned}
\tag{B.4}
$$

where,

$$
\begin{aligned}
A_0 &= \frac{a_2^2 + 4a_1^2 + 6a_0^2 + 4a_{-1}^2 + a_{-2}^2}{16\ \sum_l a_l^2}, \\
A_1 &= \frac{a_2^2 + 2a_1^2 - 2a_{-1}^2 - a_{-2}^2}{16\ \sum_l a_l^2}, \\
A_2 &= \frac{a_2^2 - 2a_0^2 + a_{-2}^2}{16\ \sum_l a_l^2}, \\
A_3 &= \frac{a_2^2 - 2a_1^2 + 2a_{-1}^2 - a_{-2}^2}{16\ \sum_l a_l^2}, \\
A_4 &= \frac{a_2^2 - 4a_1^2 + 6a_0^2 - 4a_{-1}^2 + a_{-2}^2}{16\ \sum_l a_l^2}, \\
\beta &= \frac{a_1^2 - a_{-1}^2}{\sum_l a_l^2}, \qquad \delta = \frac{a_0^2}{\sum_l a_l^2}
\end{aligned}
\tag{B.5}
$$

and

$$
a_2^2 = \frac{5}{4\pi} \sum_{l_1} |\mathcal{M}^2_{l_1,l_1-2}|^2 \qquad |l_1| \le s_1, \quad |l_1 - 2| \le s_2
$$

$$
\begin{aligned}
a_1^2 &= \frac{5}{4\pi}\sum_{l_1}|\mathcal{M}^2_{l_1,l_1-1}|^2 \qquad |l_1|\le s_1,\quad |l_1-1|\le s_2\\
a_0^2 &= \frac{5}{4\pi}\sum_{l_1}|\mathcal{M}^2_{l_1,l_1}|^2 \qquad |l_1|\le \min s_1, s_2\\
a_{-1}^2 &= \frac{5}{4\pi}\sum_{l_1}|\mathcal{M}^2_{l_1,l_1+1}|^2 \qquad |l_1|\le s_1,\quad |l_1+1|\le s_2\\
a_{-2}^2 &= \frac{5}{4\pi}\sum_{l_1}|\mathcal{M}^2_{l_1,l_1+2}|^2 \qquad |l_1|\le s_1,\quad |l_1+2|\le s_2.
\end{aligned} \tag{B.6}
$$

C. Helicity amplitudes and the analysing power

In this section we give expressions for the helicity amplitudes pertaining to 2–body decay processes of spin–$\frac{1}{2}$ and spin–1 particles. The expressions are derived for a general dimension-4 effective operator describing the coupling of the particles. This will permit to give the different analysing power coefficients.

We take the mass of the mother particle to be m and that of daughters to be m_1 and m_2, the polar and azimuthal angle belongs to the first particle with mass m_1. The energy and the momentum of the daughter particles are given as

$$
\begin{aligned}
E_1 &= \frac{m^2+m_1^2-m_2^2}{2m}, \qquad E_2 = \frac{m^2+m_2^2-m_1^2}{2m},\\
p &= \frac{\sqrt{((m+m_2)^2-m_1^2)((m+m_1)^2-m_2^2)}}{2m}
\end{aligned} \tag{C.1}
$$

from 2–body decay kinematics. Below we discuss the 2–body decay of a fermion and a vector boson into two massive particles.

C.1 Decay: $|\frac{1}{2},\lambda\rangle \to |\frac{1}{2},\lambda_1\rangle + |1,\lambda_2\rangle$

For this decay the helicity for the fermion $\lambda = \pm 1/2$ will be denoted as $\lambda = \pm 1/2$ and for the bosons $\lambda = \pm 1$ as $\lambda = \pm$ such that the helicity $M(\lambda,\lambda_1,\lambda_2)$ writes as $M(+,+,+) = M(+\frac{1}{2},+\frac{1}{2},+1)$.
The decay vertex it taken to be $\bar{f}_1\gamma^\mu\ (C_L P_L + C_R P_R) f_2 V_\mu$ with real $C_{L,R}$ and the amplitudes are listed below in the rest frame of the decaying particle:

$$
\begin{aligned}
M(+,+,+) &= \left[-(C_L\ P_1^- - C_R\ P_1^+)\right]\ e^{\frac{+i\phi}{2}}\ \left(-\sin\frac{\theta}{2}\right)\\
M(+,+,0) &= \left[-(C_L\ P_1^-P_2^- - C_R\ P_1^+P_2^+)\right]\ e^{\frac{+i\phi}{2}}\ \left(+\cos\frac{\theta}{2}\right)\\
M(+,+,-) &= 0\\
M(+,-,+) &= 0\\
M(+,-,0) &= \left[+(C_L\ P_1^+P_2^+ - C_R\ P_1^-P_2^-)\right]\ e^{\frac{+i\phi}{2}}\ \left(-\sin\frac{\theta}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
M(+,-,-) &= \left[+(C_L\ P_1^+ - C_R\ P_1^-)\right]\ e^{\frac{+i\phi}{2}}\ \left(+\cos\frac{\theta}{2}\right)\\
M(-,+,+) &= \left[-(C_L\ P_1^- - C_R\ P_1^+)\right]\ e^{\frac{-i\phi}{2}}\ \left(+\cos\frac{\theta}{2}\right)\\
M(-,+,0) &= \left[-(C_L\ P_1^- P_2^- - C_R\ P_1^+ P_2^+)\right]\ e^{\frac{-i\phi}{2}}\ \left(+\sin\frac{\theta}{2}\right)\\
M(-,+,-) &= 0\\
M(-,-,+) &= 0\\
M(-,-,0) &= \left[+(C_L\ P_1^+ P_2^+ - C_R\ P_1^- P_2^-)\right]\ e^{\frac{-i\phi}{2}}\ \left(+\cos\frac{\theta}{2}\right)\\
M(-,-,-) &= \left[+(C_L\ P_1^+ - C_R\ P_1^-)\right]\ e^{\frac{-i\phi}{2}}\ \left(+\sin\frac{\theta}{2}\right)
\end{aligned}
\tag{C.2}
$$

Here, the terms in the square brackets are the reduced matrix elements, $(\mathcal{M}^s_{\lambda_1,\lambda_2}/\sqrt{2\pi})$ and the $d^s_{\lambda,\lambda_1-\lambda_2}$ functions are enclosed in round brackets. The symbols $P^{\pm}_{1,2}$ are defined as

$$
P_1^{\pm} = \sqrt{m}\ \ \frac{E_1 + m_1 \pm p}{\sqrt{E_1 + m_1}}, \qquad P_2^{\pm} = \frac{1}{\sqrt{2}}\sqrt{\frac{E_2 \pm p}{E_2 \mp p}}. \tag{C.3}
$$

Using the expressions of the reduced matrix elements, the analysing power α for this decay can be written as

$$
\alpha = \frac{(C_R^2 - C_L^2)(1 - x_1^2 - 2x_2^2)\sqrt{1 + (x_1^2 - x_2^2)^2 - 2(x_1^2 + x_2^2)}}{(C_R^2 + C_L^2)(1 - 2x_1^2 + x_2^2 + x_1^2 x_2^2 + x_1^4 - 2x_2^4) - 12 C_L C_R x_1 x_2^2}\ , \tag{C.4}
$$

where $x_i = m_i/m$. For the decay of top quark, $t \to bW$, with $m_1 = m_b = 0$ within the SM we have $C_R = 0$ leading to $\alpha = -(1 - 2x_2^2)/(1 + 2x_2^2) \sim -0.38$.

C.2 Decay: $|\frac{1}{2}, \lambda\rangle \to |\frac{1}{2}, \lambda_1\rangle + |0, 0\rangle$

As done in the previous section the helicity for the fermions $\lambda = \pm 1/2$ will be denoted as $\lambda = \pm 1/2$ such that the helicity $M(\lambda, \lambda_1$ writes as $M(+,+)$ for $M(+\frac{1}{2}, +\frac{1}{2})$.

For this decay the helicity amplitudes, $M(\lambda, \lambda_1) = M(+,+) = M(+\frac{1}{2}, +\frac{1}{2})$. The decay vertex it taken to be $\bar{f}_1 \gamma^\mu\ (C_L P_L + C_R P_R) f_2\ S$ with complex $C_{L,R}$ and all the amplitudes are listed below:

$$
\begin{aligned}
M(+,+) &= \left[\frac{C_R\ P_1^- + C_L\ P_1^+}{\sqrt{2}}\right]\ e^{\frac{+i\phi}{2}}\ \left(+\cos\frac{\theta}{2}\right)\\
M(+,-) &= \left[\frac{C_R\ P_1^+ + C_L\ P_1^-}{\sqrt{2}}\right]\ e^{\frac{+i\phi}{2}}\ \left(-\sin\frac{\theta}{2}\right)\\
M(-,+) &= \left[\frac{C_R\ P_1^- + C_L\ P_1^+}{\sqrt{2}}\right]\ e^{\frac{-i\phi}{2}}\ \left(+\sin\frac{\theta}{2}\right)
\end{aligned}
$$

$$M(-,-) = \left[\frac{C_R\ P_1^+ + C_L\ P_1^-}{\sqrt{2}}\right]\ e^{\frac{-i\phi}{2}}\ \left(+\cos\frac{\theta}{2}\right) \tag{C.5}$$

Here, the reduced matrix elements, $(\mathcal{M}^s_{\lambda_1,0}/\sqrt{2\pi})$, are given in square brackets and the d^s_{λ,λ_1} functions in the round brackets. The symbols $P_1^\pm$ are same as in Eq.(C.3). Using the expressions of reduced matrix elements we get the expression for α, the analysing power of the spin–$\frac{1}{2}$ particle, as

$$\alpha = \frac{-(|C_R|^2 - |C_L|^2)\sqrt{1 + (x_1^2 - x_2^2)^2 - 2(x_1^2 + x_2^2)}}{(|C_R|^2 + |C_L|^2)(1 + x_1^2 - x_2^2) + 4x_1\Re(C_L C_R^*)}\quad , \tag{C.6}$$

where $x_i = m_i/m$. Thus we need the scalar to have parity violating couplings, $|C_L| \neq |C_R|$, for the analysing power to be non-zero. However, for a neutral scalar, say the neutral Higgs boson of the MSSM, we have $C_L = C_R^*$ in other words $|C_L| = |C_R|$ leading to $\alpha = 0$. The same occurs in a CP conserving MSSM with any of the neutral Higgs boson. Thus we should chose processes involving squarks for spin measurement in the decay of gauginos.

C.3 Decay: $|1, \lambda\rangle \to |\frac{1}{2}, \lambda_1\rangle + |\frac{1}{2}, \lambda_2\rangle$

We take the same convention as in C.1 with the same operator for the interaction. We find

$$\begin{aligned}
M(+,+,+) &= \left[-i\frac{C_R\ p_1^+ p_2^- + C_L\ p_1^- p_2^+}{2}\right]\ e^{+i\phi}\ \left(\frac{-1}{\sqrt{2}}\sin\theta\right)\\
M(+,+,-) &= \left[+i\frac{C_R\ p_1^+ p_2^+ + C_L\ p_1^- p_2^-}{\sqrt{2}}\right]\ e^{+i\phi}\ \left(\cos^2\frac{\theta}{2}\right)\\
M(+,-,+) &= \left[-i\frac{C_R\ p_1^- p_2^- + C_L\ p_1^+ p_2^+}{\sqrt{2}}\right]\ e^{+i\phi}\ \left(\sin^2\frac{\theta}{2}\right)\\
M(+,-,-) &= \left[+i\frac{C_R\ p_1^- p_2^+ + C_L\ p_1^+ p_2^-}{2}\right]\ e^{+i\phi}\ \left(\frac{-1}{\sqrt{2}}\sin\theta\right)\\
M(0,+,+) &= \left[-i\frac{C_R\ p_1^+ p_2^- + C_L\ p_1^- p_2^+}{2}\right]\ (\cos\theta)\\
M(0,+,-) &= \left[+i\frac{C_R\ p_1^+ p_2^+ + C_L\ p_1^- p_2^-}{\sqrt{2}}\right]\ \left(\frac{+1}{\sqrt{2}}\sin\theta\right)\\
M(0,-,+) &= \left[-i\frac{C_R\ p_1^- p_2^- + C_L\ p_1^+ p_2^+}{\sqrt{2}}\right]\ \left(\frac{-1}{\sqrt{2}}\sin\theta\right)\\
M(0,-,-) &= \left[+i\frac{C_R\ p_1^- p_2^+ + C_L\ p_1^+ p_2^-}{2}\right]\ (\cos\theta)\\
M(-,+,+) &= \left[-i\frac{C_R\ p_1^+ p_2^- + C_L\ p_1^- p_2^+}{2}\right]\ e^{-i\phi}\ \left(\frac{+1}{\sqrt{2}}\sin\theta\right)\\
M(-,+,-) &= \left[+i\frac{C_R\ p_1^+ p_2^+ + C_L\ p_1^- p_2^-}{\sqrt{2}}\right]\ e^{-i\phi}\ \left(\sin^2\frac{\theta}{2}\right)
\end{aligned}$$

$$M(-,-,+) = \left[-i\frac{C_R\ p_1^- p_2^- + C_L\ p_1^+ p_2^+}{\sqrt{2}}\right]\ e^{-i\phi}\ \left(\cos^2\frac{\theta}{2}\right)$$
$$M(-,-,-) = \left[+i\frac{C_R\ p_1^- p_2^+ + C_L\ p_1^+ p_2^-}{2}\right]\ e^{-i\phi}\ \left(\frac{+1}{\sqrt{2}}\sin\theta\right) \tag{C.7}$$

In Eq. C.7 the terms in the square bracket stand for the reduced matrix elements $(\mathcal{M}^s_{\lambda_1,\lambda_2}\sqrt{\frac{3}{4\pi}})$, the terms in the round brackets are the $d^s_{\lambda,\lambda_1-\lambda_2}$ functions. The symbols $p^\pm_{1,2}$ are defined as

$$p_1^\pm = \frac{E_1 + m_1 \pm p}{\sqrt{E_1+m_1}}, \quad p_2^\pm = \frac{E_2 + m_2 \pm p}{\sqrt{E_2+m_2}}, \tag{C.8}$$

Using the expressions for a^s_l and reduced matrix element we get expressions for two parameter α and δ as

$$\alpha = \frac{2(C_R^2 - C_L^2)\sqrt{1 + (x_1^2 - x_2^2)^2 - 2(x_1^2 + x_2^2)}}{12C_L C_R x_1 x_2 + (C_R^2 + C_L^2)[2 - (x_1^2 - x_2^2)^2 + (x_1^2 + x_2^2)]}, \tag{C.9}$$
$$\delta = \frac{4C_L C_R x_1 x_2 + (C_R^2 + C_L^2)[(x_1^2 + x_2^2) - (x_1^2 - x_2^2)^2]}{12C_L C_R x_1 x_2 + (C_R^2 + C_L^2)[2 - (x_1^2 - x_2^2)^2 + (x_1^2 + x_2^2)]}, \tag{C.10}$$

where $x_i = m_i/m$.
If the final state fermions are massless, $x_1 \to 0, x_2 \to 0$, one obtains $\alpha \to (C_R^2 - c_L^2)/(C_R^2 + C_L^2)$ and $\delta \to 0$. This is the case for the decay of W and Z bosons into massless fermions. Further, for the decay of Ws, within the SM we have $C_R = 0$ hence $\alpha = -1$.

C.4 Decay: $|1,\lambda\rangle \to |1,\lambda_1\rangle + |0,0\rangle$

For this decay the helicity amplitudes, $M(\lambda,\lambda_1) = M(+,+) = M(+1,+1)$. The decay vertex it taken to be $C_{VVS}g_{\mu\nu}V^\mu V_1^\nu$ with real C_{VVS} and the helicity amplitudes are given by:

$$M(+,+) = [-C_{VVS}]\ e^{+i\phi}\ \left(\cos^2\frac{\theta}{2}\right)$$
$$M(+,0) = \left[-C_{VVS}\frac{E_1}{m_1}\right]\ e^{+i\phi}\ \left(\frac{-\sin\theta}{\sqrt{2}}\right)$$
$$M(+,-) = [-C_{VVS}]\ e^{+i\phi}\ \left(\sin^2\frac{\theta}{2}\right)$$
$$M(0,+) = [-C_{VVS}]\ \left(\frac{\sin\theta}{\sqrt{2}}\right)$$
$$M(0,0) = \left[-C_{VVS}\frac{E_1}{m_1}\right]\ (\cos\theta)$$
$$M(0,-) = [-C_{VVS}]\ \left(\frac{-\sin\theta}{\sqrt{2}}\right)$$

$$
\begin{aligned}
M(-,+) &= [-C_{VVS}]\ e^{-i\phi}\ \left(\sin^2\frac{\theta}{2}\right) \\
M(-,0) &= \left[-C_{VVS}\frac{E_1}{m_1}\right]\ e^{-i\phi}\ \left(\frac{\sin\theta}{\sqrt{2}}\right) \\
M(-,-) &= [-C_{VVS}]\ e^{-i\phi}\ \left(\cos^2\frac{\theta}{2}\right)
\end{aligned}
\tag{C.11}
$$

This leads to $a^1_1 = a^1_{-1}$ hence $\alpha = 0$. δ is given by

$$
\delta = \frac{(1+x_1^2-x_2^2)^2}{1+(x_1^2-x_2^2)^2+2(5x_1^2-x_2^2)}.
\tag{C.12}
$$

Such decays occur the models of extra-dimensions, for example, $W^{(1)} \to W^{(0)}h$, $Z^{(1)} \to Z^{(0)}h$ etc.

C.5 Decay: $|1,\lambda\rangle \to |1,\lambda_1\rangle + |1,\lambda_2\rangle$

For this decay the helicity amplitudes, $M(\lambda,\lambda_1,\lambda_2) = M(+,+,+) = M(+1,+1,+1)$. The decay vertex is taken to be $C_{VVV}T_{\mu\nu\rho}V^\mu V_1^\nu V_2^\rho$ where C_{VVV} is real. We only assume here a standard *gauge* tri-linear coupling

$$
T_{\mu\nu\rho} = [g_{\mu\nu}(q-p_1)_\rho + g_{\nu\rho}(p_1-p_2)_\mu + g_{\rho\mu}(p_2-q)_\nu]
$$

with q being the 4-momentum of the mother particle and $p_{1,2}$ is the 4-momentum of the daughter particles. All momenta are assumed incoming at the interaction vertex. With this notation the non-zero helicity amplitudes are given by:

$$
\begin{aligned}
M(+,+,+) &= [2C_{VVV}\ p]\ e^{+i\phi}\ \left(\frac{-\sin\theta}{\sqrt{2}}\right) \\
M(+,0,0) &= \left[-2C_{VVV}\ p\ \frac{E_1E_2+p^2}{m_1m_2}\right]\ e^{+i\phi}\ \left(\frac{-\sin\theta}{\sqrt{2}}\right) \\
M(+,-,-) &= [2C_{VVV}\ p]\ e^{+i\phi}\ \left(\frac{-\sin\theta}{\sqrt{2}}\right) \\
M(0,+,+) &= [2C_{VVV}\ p]\ (\cos\theta) \\
M(0,0,0) &= \left[-2C_{VVV}\ p\ \frac{E_1E_2+p^2}{m_1m_2}\right]\ (\cos\theta) \\
M(0,-,-) &= [2C_{VVV}\ p]\ (\cos\theta) \\
M(-,+,+) &= [2C_{VVV}\ p]\ e^{-i\phi}\ \left(\frac{-\sin\theta}{\sqrt{2}}\right) \\
M(-,0,0) &= \left[-2C_{VVV}\ p\ \frac{E_1E_2+p^2}{m_1m_2}\right]\ e^{-i\phi}\ \left(\frac{-\sin\theta}{\sqrt{2}}\right) \\
M(-,-,-) &= [2C_{VVV}\ p]\ e^{-i\phi}\ \left(\frac{-\sin\theta}{\sqrt{2}}\right)
\end{aligned}
\tag{C.13}
$$

This leads to $a^1_1 = a^1_{-1} = 0$ and hence $\alpha = 0$ and $\delta = 1$. Example of such decays, in the models of extra-dimensions, are $W^{(1)} \to W^{(0)} Z^{(0)}$, $W^{(1)} \to W^{(0)} \gamma^{(0)}$ etc.

Above we saw that the parameters α and δ have a simple expressions in terms of masses and couplings of the particles involved. This can be simply added to a spectrum generation code, such as `SOFTSUSY` [35], `SuSpect` [36], `SPheno` [37] for SUSY models, and one can quickly know which decay channel is the best for estimation of the particle's spin.

D. Higher spin particle disguising as lower spin particle

If a higher spin particle is produced as a decay product of the lower spin particle, its spin orientations are restricted. This makes the particle appear as of a lower spin in frame M see Sec. 4.2. Since the total differential rate is the product of production and decay density matrices,

$$d\sigma = \frac{1}{2I}\ \rho^s(l,l')\ \times\ \Gamma^s(l,l')\ \ d\Phi_n\ .$$

Thus, for the decay distribution to have a $2s\phi$ modulation, we must have $\rho^s(s,-s) = \rho^{s*}(-s,s) \neq 0$. Now if this spin s particle were produced in the decay reaction $|j,m\rangle \to |s_1,l_1\rangle + |s,l\rangle$, then the production density matrix is given by

$$\begin{aligned}\rho^s(l,l') &= \sum_{m,l_1} M^{jm}_{l_1,l}\ M^{jm*}_{l_1,l'} \\ &= \left(\frac{2j+1}{4\pi}\right) e^{i(l-l')\phi} \sum_{m,l_1} d^j_{m,l_1-l}\ d^j_{m,l_1-l'}\ \ \mathcal{M}^j_{l_1,l}\mathcal{M}^{j*}_{l_1,l'}. \end{aligned} \tag{D.1}$$

Thus we have extreme off-diagonal term given by

$$\rho^s(s,-s) = \left(\frac{2j+1}{4\pi}\right) e^{i2s\phi} \sum_{m,l_1} d^j_{m,l_1-s}\ d^j_{m,l_1+s}\ \ \mathcal{M}^j_{l_1,l}\mathcal{M}^{j*}_{l_1,l'}. \tag{D.2}$$

For this to be non-zero, we must have

$$|l_1 - s| \leq j \quad \text{and} \quad |l_1 + s| \leq j$$

for at least one value of l_1. However, this condition is never satisfied (for any l_1) when we have $s > j$, i.e. $\rho^s(s,-s) = 0$ for $s > j$. This leads to the absence of the highest mode in the ϕ distribution in frame M. This is numerically demonstrated for the sample of W boson production from the decay of t-quark. Since the helicities are invariant only under the boost along the momentum (which does not changes the direction of the momentum), the density matrix goes through a similarity transformation when boosted in any other direction. Thus in frame F, in general one can have a non-zero value for $\rho^s(s,-s)$ and hence the $2s\phi$ modulation of the azimuthal distribution.

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